MAT 110 WORKSHOP

Updated Fall 2018
UNIT 2: COUNTING AND PROBABILITY

Introduction
Introduction to Counting Methods

• Count elements in a set systematically.
• Use tree diagrams to represent counting situations graphically.
• Use counting techniques to solve applied problems.
Tree Diagrams

- Example: How many ways can three coins be flipped?
- Solution: Let’s assume we are flipping a penny, a nickel, and a dime. A tree diagram is a handy way to illustrate the possibilities.

First illustrate the possibilities for flipping the penny.
Next illustrate the possibilities for the nickel after having already flipped the penny.
Tree Diagrams

Now illustrate the possibilities for the dime after having already flipped the penny and the nickel. We can trace eight branches that indicate the ways the three coins can be flipped: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.
Tree Diagrams

• Example: The director at a local TV station wants to fill three commercial spots using promos for the latest albums by singers (J)ordin, (T)aylor, and (C)arrie. In how many ways can these spots be filled if repetition is allowed?
Tree Diagrams

The tree diagram illustrates all possibilities. We see that there are 27 ways to fill the promo slots.
Tree Diagrams

The tree diagram illustrates all possibilities if repetition is not allowed. We see that there are 6 ways to fill the promo slots.
Definitions

**Fundamental Counting Principle:** If we want to perform a series of tasks and the first task can be done in \(a\) ways, the second task can be done in \(b\) ways, the third can be done in \(c\) ways, and so on, then all the tasks can be done in \(a \cdot b \cdot c \ldots\) ways.

**Permutations:** an *ordering* of distinct objects in a straight line. The number of ways to order \(r\) objects from a set of \(n\) objects is denoted \(P(n, r)\).

\[
P(n, r) = \frac{n!}{(n-r)!}
\]

**Factorial Notation:** \(n! = n \times (n - 1) \times (n - 2) \times \ldots \times 3 \times 2 \times 1\). By definition, \(0! = 1\).

**Combinations:** choosing \(r\) objects from a set of \(n\) objects (order does NOT matter). Notation \(C(n, r)\) or \(\binom{n}{r}\). Read as “\(n\) choose \(r\).”

\[
C(n, r) = \frac{n!}{r!(n-r)!}
\]

**Probability:** the proportion of times the outcome would occur in a very long series of repetitions.

**Sample Space:** the set of all possible outcomes.

**Event:** a subset of a sample space.

\(P(E)\): read as “the probability of event \(E\).” \(0 \leq P(E) \leq 1\). Also, the sum of the probabilities of all the outcomes in a sample space must be 1.

**Complement:** The complement of Event \(E\) is the set of all outcomes not in \(E\). It is denoted by \(\bar{E}\).

**Union:** If \(S\) is a sample space and \(E\) and \(F\) are events, then the union of \(E\) or \(F\) is the set of all outcomes in \(E\) or \(F\). It is denoted by \(E \cup F\).

**Intersection:** If \(S\) is a sample space and \(E\) and \(F\) are events, then the intersection of \(E\) and \(F\) is the set of all outcomes in both \(E\) and \(F\). It is denoted by \(E \cap F\).
Fundamental Counting Principle (FCP)

If we want to perform a series of tasks and the first task can be done in $a$ ways, the second can be done in $b$ ways, the third can be done in $c$ ways, and so on, then all the tasks can be done in $(a \cdot b \cdot c \cdot ...)\) ways.

Example: How many ways can three six-sided dice (red, green, blue) be rolled together?

Solution: The red die has 6 possible outcomes, \{1,2,3,4,5,6\}. The green and blue dice also have 6 possible outcomes. The three dice can be rolled in

$$6 \times 6 \times 6 = 216 \text{ ways}.$$
The Fundamental Counting Principle

• Example: A summer intern wants to vary his outfit by wearing different combinations of coats, pants, shirts, and ties. If he has three sports coats, five pairs of pants, seven shirts, and four ties, how many different ways can he select an outfit consisting of a coat, pants, shirt, and tie?
The Fundamental Counting Principle

Solution: The interns options are

<table>
<thead>
<tr>
<th>Task</th>
<th>Number of Ways to Perform Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select coat</td>
<td>3</td>
</tr>
<tr>
<td>Select pants</td>
<td>5</td>
</tr>
<tr>
<td>Select shirt</td>
<td>7</td>
</tr>
<tr>
<td>Select tie</td>
<td>4</td>
</tr>
</tbody>
</table>

\[3 \times 5 \times 7 \times 4 = 420 \text{ outfits}\]

number of coats \(\times\) times number of pants \(\times\) times number of shirts \(\times\) times number of ties
Slot Diagrams

A useful technique for solving problems involving various tasks is to draw a series of blank spaces to keep track of the number of ways to do each task. We will call such a figure a *slot diagram*.
Slot Diagrams

- Example: A security keypad uses five digits (0 to 9) in a specific order. How many different keypad patterns are possible if any digit can be used in any position and repetition is allowed?

- Solution: The slot diagram indicates there are $10 \times 10 \times 10 \times 10 \times 10 = 100,000$ possibilities.
Example: A college class has 10 students. Louise must sit in the front row next to her tutor. If there are six chairs in the first row of the classroom, how many different ways can students be assigned to sit in the first row?

First, determine our tasks:

- Task 1 – Assign Louise and her tutor a pair of chairs next to one another.
- Task 2 – Arrange Louise and her tutor in the pair of chairs.
- Task 3 – Assign students to the remaining four chairs.
Task 1

**Task 1:** There are five ways to assign 2 seats to Louise and her tutor.

<table>
<thead>
<tr>
<th>Seat 1</th>
<th>Seat 2</th>
<th>Seat 3</th>
<th>Seat 4</th>
<th>Seat 5</th>
<th>Seat 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>and</td>
<td>T</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seat 1</th>
<th>Seat 2</th>
<th>Seat 3</th>
<th>Seat 4</th>
<th>Seat 5</th>
<th>Seat 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>L</td>
<td>and</td>
<td>T</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seat 1</th>
<th>Seat 2</th>
<th>Seat 3</th>
<th>Seat 4</th>
<th>Seat 5</th>
<th>Seat 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>X</td>
<td>L</td>
<td>and</td>
<td>T</td>
<td>X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seat 1</th>
<th>Seat 2</th>
<th>Seat 3</th>
<th>Seat 4</th>
<th>Seat 5</th>
<th>Seat 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>X</td>
<td>X</td>
<td>L</td>
<td>and</td>
<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seat 1</th>
<th>Seat 2</th>
<th>Seat 3</th>
<th>Seat 4</th>
<th>Seat 5</th>
<th>Seat 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>L</td>
<td>and</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seat 1</th>
<th>Seat 2</th>
<th>Seat 3</th>
<th>Seat 4</th>
<th>Seat 5</th>
<th>Seat 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>L</td>
</tr>
</tbody>
</table>
Tasks 2 and 3

Task 2: There are two ways that Louise and her tutor can sit in their seats—Louise sits either on the right or the left.

Task 3: The remaining four seats are to be filled by four of the eight students left; thus, we have eight students for the first remaining seat, seven for the second seat, and so on.

\[ 5 \times 2 \times 8 \times 7 \times 6 \times 5 = 16,800. \]

- five ways to choose the two seats for Louise and her tutor
- two ways to seat Louise and the tutor in the two seats
- number of ways to seat four of eight students in the remaining four seats
- total number possibilities
License plates in Florida have the form A24BSDE; that is, a letter followed by 2 digits followed by 4 more letters.

A) Number of possible license plates

B) Griff would like a plate that ends in MWSU. How many such plates are there?
Handling Special Conditions

A) There are 26 letters in the alphabet and 10 digits, you need 5 letters and 2 numbers so:
\[26 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 1188137600\] possible plates

B) Now you only need 1 letter and 2 numbers so:
\[26 \cdot 10 \cdot 10 = 2600\] possible plates
Example: In how many different ways can the three wheels of the slot machine depicted below stop?
Solution: Using the fundamental counting principle, we see that the total number of different ways the three wheels can stop is

\[ 20 \cdot 20 \cdot 20 = 8,000. \]
Applying the Fundamental Counting Principle to Gambling

- Example: One payoff for a slot machine with the wheels shown below is for cherries on wheel 1 and no cherries on the other wheels. In how many different ways can this happen?

(continued on next slide)
Applying the Fundamental Counting Principle to Gambling

- Solution: By the fundamental counting principle, we see that the total number of ways we can get cherries only on the first wheel is \(2 \cdot 15 \cdot 12 = 360\).
Applying the Fundamental Counting Principle to Gambling

- Example: The largest payoff for a slot machine with the wheels shown below is for three bars. In how many ways can three bars be obtained?

(continued on next slide)
Applying the Fundamental Counting Principle to Gambling

• Solution: By the fundamental counting principle, a bar on the first, second, and third wheel can occur in

\[2 \cdot 3 \cdot 1 = 6 \text{ ways}.\]
Permutations and Combinations

• Calculate the number of permutations of \( n \) objects taken \( r \) at a time.

• Use factorial notation to represent the number of permutations of a set of objects.

• Apply the theory of permutations and combinations to solve counting problems.
**Permutations**

**Definition** A permutation is an ordering of distinct objects in a straight line. If we select \( r \) different objects from a set of \( n \) objects and arrange them in a straight line, this is called a permutation of \( n \) objects taken \( r \) at a time. The number of permutations of \( n \) objects taken \( r \) at a time is denoted by \( P(n, r) \).

**Formula for Computing** \( P(n, r) \)

\[
P(n, r) = \frac{n!}{(n - r)!}
\]
Example: The 12-person theater group wishes to select one person to direct a play, a second to supervise the music, and a third to handle publicity, tickets, and other administrative details. In how many ways can the group fill these positions?

Solution: This is a permutation of selecting 3 people from 12.

\[
P(12, 3) = \frac{12!}{(12 - 3)!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1,320
\]
Permutations

• **Example:** How many permutations are there of the letters $a, b, c,$ and $d$? Write the answer using $P(n, r)$ notation.

• **Solution:** We write the letters $a, b, c,$ and $d$ in a line without repetition, so $abcd$ and $bcad$ are two such permutations.
Permutations

The slot diagram indicates there are $4 \times 3 \times 2 \times 1 = 24$ possibilities.

This can be written more succinctly as

$$P(4, 4) = 24.$$
Permutations

• Example: How many permutations are there of the letters \(a, b, c, d, e, f,\) and \(g\) if we take the letters three at a time? Write the answer using \(P(n, r)\) notation.

• Solution: The slot diagram indicates there are \(7 \times 6 \times 5 = 210\) possibilities. This can be written in permutation notation as shown.

\[
P(7, 3) = 210
\]
The math club needs to select a President, Vice President, Treasurer and Secretary from their 28 members. How many ways can they do this?
You have 28 people in your club but you only need to choose 4 of them. Order DOES matter since each person is being selected for a specific position so you have to use permutation.

\[ P(28, 4) = 491400 \] Number of possible leadership committees
Factorial Notation

**Definition** If $n$ is a counting number, the symbol $n!$, called $n$ factorial, stands for the product $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots \cdot 2 \cdot 1$. We define $0! = 1$. 

- **Example:** Compute $(8 - 3)!$.
- **Solution:** We work inside parentheses first.

\[
(8 - 3)! = 5! \\
= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\
= 120
\]
Factorial Notation

• Example: Compute \( \frac{8!}{5!3!} \).

• Solution:

\[
\frac{8!}{5!3!} = \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1\cdot3\cdot2\cdot1} = 8\cdot7 = 56.
\]

Cancel 5!, 
Cancel 3!, which equals 6.
Combinations

**Formula for Computing $C(n, r)$**  If we choose $r$ objects from a set of $n$ objects, we say that we are forming a combination of $n$ objects taken $r$ at a time. The notation $C(n, r)$ denotes the number of such combinations.† Also,

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{r! \cdot (n-r)!}.$$
Combinations

• Example: How many four-person committees can be formed from a set of 10 people?

• Solution: Order is not important, so it is clear that this is a combination problem.

\[ C(10, 4) = \frac{10!}{4! \cdot (10 - 4)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot x}{4 \cdot 3 \cdot 2 \cdot x \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot x} = 210 \]
Combination

• Example: In the game of poker, five cards are drawn from a standard 52-card deck. How many different poker hands are possible?

• Solution:

\[ C(52, 5) = \frac{52!}{5!47!} = \frac{13 \cdot 17 \cdot 10 \cdot 24}{8 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2,598,960 \]
Combinations

• Example: In the game of Euchre, a hand consists of 5 cards drawn from a deck of 9, 10, Jack, Queen, King, Ace from each suit. How many different Euchre hands are possible?

Euchre deck: Special Euchre decks are available, or the standard 52-card pack can be stripped to make a deck of 24 cards (A, K, Q, J, 10, 9 of each suit)
Euchre Deck

• In the game of euchre, the deck consists of the 9, 10, jack, queen, king and ace of each suit. Players are dealt a five card hand.

• What is the probability that a player is dealt 3 hearts?
There are 6 hearts in a Euchre deck and we want to pick 3 of them. \( C(6, 3) = 20 \). We still need to pick 2 card from the remaining 18 non-heart cards to complete our 5 card hand. \( C(18, 2) = 153 \). The number of ways to have 3 hearts in a 5 card hand is \( 20 \times 153 = 3060 \).

The number of total possible outcomes is \( C(24, 5) = 42504 \).

So the probability of getting 3 hearts is \( \frac{3060}{42504} \) or \( 0.072 \) or 7.2\%.
Combining Counting Methods

• Example: Two men and two women from a firm will attend a conference. If the firm has ten men and nine women, in how many different ways can the conference attendees be selected?

• Solution: The answer is not $C(19, 4)$ since this includes options like four men and no woman being sent to the conference.
Combining Counting Methods

**Stage 1:** Select the two women from the nine available.

\[
C(9, 2) = \frac{9!}{2!7!} = 36 \text{ ways}
\]

**Stage 2:** Select the two men from the ten available.

\[
C(10, 2) = \frac{10!}{2!8!} = 45 \text{ ways}
\]

Thus, choosing the women and then choosing the men can be done in \(36 \cdot 45 = 1,620\) ways.
Example: A 16-member organization wishes to choose a committee consisting of a president, a vice president, and a three-member executive board. In how many different ways can this committee be formed?

Solution: We will count this in two stages:
(a) choosing the president and vice president from the organization members,
(b) choosing an executive board from the remaining members.

(continued on next slide)
Stage 1: Choose the president and vice president. This can be done in $P(16, 2)$ ways.

Stage 2: Select the executive board. This can be done in $C(14, 3)$ ways.

Total: $P(16, 2) \times C(14, 3) = 87,360$ ways.
Examples

1. $6! = $

2. $\frac{6!}{3!3!} = $

3. $P(8, 3) =$

4. $C(8, 3) =$

5. $\binom{7}{2} =$

6. $(8 - 6)! = $
Examples

1. $6! = 720$

2. $\frac{6!}{3!3!} = 20$

3. $P(8, 3) = 336$

4. $C(8, 3) = 56$

5. $\binom{7}{2} = 21$

6. $(8 - 6)! = 2$
**Sample Space and Events**

**Definitions** An experiment is any observation of a random phenomenon. The different possible results of the experiment are called outcomes. The set of all possible outcomes for an experiment is called a sample space.

**Definition** In probability theory, an event is a subset of the sample space.

**Some Good Advice**

Although we usually describe events verbally, you should remember that an event is always a subset of the sample space. You can use the verbal description to identify the set of outcomes that make up the event.
DEFINITIONS The **probability of an outcome** in a sample space is a number between 0 and 1 inclusive. The sum of the probabilities of all the outcomes in the sample space must be 1. The **probability of an event** $E$, written $P(E)$, is defined as the sum of the probabilities of the outcomes that make up $E$.

EMPIRICAL ASSIGNMENT OF PROBABILITIES If $E$ is an event and we perform an experiment several times, then we estimate the probability of $E$ as follows:

$$P(E) = \frac{\text{the number of times } E \text{ occurs}}{\text{the number of times the experiment is performed}}.$$  

This ratio is sometimes called the **relative frequency** of $E$. 
**BASIC PROPERTIES OF PROBABILITY** Assume that $S$ is a sample space for some experiment and $E$ is an event in $S$.

1. $0 \leq P(E) \leq 1$
2. $P(\emptyset) = 0$
3. $P(S) = 1$

**Intuitive meaning of probability.**
Probability

- Three students from an 18 student class will be selected to attend a meeting. 11 of the students are female.

- What is the probability that not all three of the students chosen to attend the meeting are female?
Probability

- \( C(11, 3) = 165 \) so there is 165 ways to pick just girls.
- \( C(18, 3) = 816 \) so there is 816 possible ways to choose out of everyone.

\[
\frac{165}{816} = .20220588
\]

so there is about a 20.2% chance of picking 3 girls. We want the probability of NOT picking 3 girls.

To find this, take \( 1 - .20220588 = .7978 \) so there is a 79.78% chance of not picking all three girls.
Sample Space and Events

- Example: The table summarizes the marital status of men and women (in thousands) in the United States in 2006. If we randomly pick a male, what is the probability that he is divorced?

<table>
<thead>
<tr>
<th></th>
<th>Now Married (except separated)</th>
<th>Widowed</th>
<th>Divorced</th>
<th>Separated</th>
<th>Never Married</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>60,955</td>
<td>2,908</td>
<td>10,818</td>
<td>2,210</td>
<td>39,435</td>
</tr>
<tr>
<td>Females</td>
<td>59,173</td>
<td>12,226</td>
<td>14,182</td>
<td>3,179</td>
<td>33,377</td>
</tr>
</tbody>
</table>
Sample Space and Events

• Solution: We are only interested in males, so we consider our sample space to be the
  \[ 60,955 + 2,908 + 10,818 + 2,210 + 39,435 = 116,326 \]
  males.

  The event, call it \( D \), is the set of 10,818 men who are divorced. Therefore, the probability that we
  would select a divorced male is

  \[
  P(D) = \frac{n(D)}{n(S)} = \frac{10,818}{116,326} \approx 0.093.
  \]
CALCULATING PROBABILITY WHEN OUTCOMES ARE EQUALLY LIKELY If $E$ is an event in a sample space $S$ with all equally likely outcomes, then the probability of $E$ is given by the formula:

$$P(E) = \frac{n(E)}{n(S)}.$$
Example: A couple wants three children. What are the arrangements of boys (B) and girls (G)? Find the probability of an individual outcome in your sample space. Genetics tells us that the probability that a baby is a boy or a girl is the same, 0.5.

→ Sample space:
{BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}

→ All eight outcomes in the sample space are equally likely.

→ The probability of each is thus 1/8.
Probability

3 three sided dice, (one red, one blue, and one green all with sides labeled 1, 2, and 3) are rolled and their sum is recorded.

A) What is the probability of rolling a sum of 4?

B) What is the probability of rolling a sum of 9?
Probability

• A) There are 3 ways to get a sum of 4 but there is a total of 27 outcomes. So the probability of rolling a sum of 4 is \( \frac{3}{27} \) or \( 0.1111 \) or 11.11%.

• B) The only way to get a sum of 9 is if every dice lands on a 3 so there is only one possible outcome of nine. So the probability of rolling a sum of 9 is \( \frac{1}{27} \) or 0.037 or 3.7%. 
Probability

• A jar contains 4 red marbles, 11 green marbles, and 6 blue marbles.

• A) What is the probability that you draw 4 green marbles in a row if you do replace the marbles after each draw?

• B) What is the probability that you draw 3 red marbles in a row if you don’t replace the marbles after each draw?

• C) What is the probability that you draw 7 blue marbles in a row if you don’t replace the marbles after each draw?
Probability

A) There is a total of 21 marbles you can choose from. 11 of those are green so you have a \( \frac{11}{21} \) chance of getting one green. In order to find 4 green when you DO replace: 
\[
\left( \frac{11}{21} \right) \cdot \left( \frac{11}{21} \right) \cdot \left( \frac{11}{21} \right) \cdot \left( \frac{11}{21} \right) = \frac{14641}{194481}
\]
or 
\[0.0753\] or 
\[7.53\%
\]

B) You have a \( \frac{4}{21} \) chance of picking one red. To find 3 red marbles in a row if you DO NOT replace:
\[
\left( \frac{4}{21} \right) \cdot \left( \frac{3}{20} \right) \cdot \left( \frac{2}{19} \right) = \frac{27}{7980}
\]
or 
\[0.003\] or 
\[0.3\%
\]
You DO NOT replace the marble after you pull it out so the number of red marbles and total marbles goes down.

C) You have a \( \frac{6}{21} \) chance of pulling a blue marble. It is impossible to pull 7 blue marbles out in a row if you DO NOT replace them because there is only 6 blue marbles.
Let $E$ and $F$ be events in a sample space $S$. Then

1. $0 \leq P(E) \leq 1$
2. $P(S) = 1$
3. $P(\emptyset) = 0$
4. $P(\bar{E}) = 1 - P(E)$
5. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
If $P(A) = .42$, $P(B) = .51$, and $P(A \cap \bar{B}) = .18$

find the following:

- $P(A)$
- $P(B)$
- $P(A \cap B)$
- $P(A \cup B)$
- $P(\bar{A})$
- $P(\bar{A} \cap B)$
- $P(\bar{A} \cup B)$
- $P(A \cap \bar{B})$
If \( P(A) = .42 \), \( P(B) = .51 \), and \( P(A \cap \overline{B}) = .18 \)
find the following:

- \( P(A) = .42 \)
- \( P(B) = .51 \)
- \( P(A \cap B) = P(A) - P(A \cap \overline{B}) = .42 - .18 = .24 \)
- \( P(A \cup B) = P(A) + P(B) - P(A \cap B) = .42 + .51 - .24 = .69 \)
- \( P(\overline{A}) = 1 - P(A) = 1 - .42 = .58 \)
- \( P(\overline{A} \cap B) = P(B) - P(A \cap B) = .51 - .24 = .27 \)
- \( P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cap B) = .58 + .51 - .27 = .82 \)
- \( P(\overline{A} \cap \overline{B}) = 1 - P(A \cap B) = 1 - .24 = .76 \)
Conditional Probability

For events $E$ and $F$, the probability of $E$ given $F$ is

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$
A diagnostic test for disease X correctly identifies the disease 89% of the time. False positives occur 12%. It is estimated that 2% of the population suffers from disease X. Suppose the test is applied to a random individual from the population.

Fill in a probability tree to describe this situation.
A diagnostic test for disease X correctly identifies the disease 89% of the time. False positives occur 12%. It is estimated that 0.5% of the population suffers from disease X. Suppose the test is applied to a random individual from the population.

- What is the percentage chance that the test will be positive?

*Note: This includes ALL positive test results, whether they are correct or not.
A diagnostic test for disease X correctly identifies the disease 89% of the time. False positives occur 12%. It is estimated that 0.5% of the population suffers from disease X. Suppose the test is applied to a random individual from the population.

- What is the percentage chance that the test will be positive?

\[ 0.02 \times 0.89 + 0.98 \times 0.12 = 0.1354 \text{ or } 13.54\% \]
A diagnostic test for disease X correctly identifies the disease 89% of the time. False positives occur 12%. It is estimated that 0.5% of the population suffers from disease X. Suppose the test is applied to a random individual from the population.

- What is the probability that given a positive result, the person has disease X?
A diagnostic test for disease X correctly identifies the disease 89% of the time. False positives occur 12%. It is estimated that 0.5% of the population suffers from disease X. Suppose the test is applied to a random individual from the population.

- What is the probability that given a positive result, the person has disease X?

\[
\text{Disease and Positive result} = \frac{.02 \times .89}{.1354} = .1315 \text{ or } 13.15\%
\]
Probability Trees

• Box A contains 8 red and 10 white marbles. Box B contains 9 red and 5 white marbles. A marble is chosen at random from Box A and its color is recorded. That marble is then placed in Box B and a marble is chosen at random from Box B and its color is recorded.
• Fill in the probability tree to describe this situation.
Box A contains 8 red and 10 white marbles. Box B contains 9 red and 5 white marbles. A marble is chosen at random from Box A and its color is recorded. That marble is then placed in Box B and a marble is chosen at random from Box B and its color is recorded.
Box A contains 8 red and 10 white marbles. Box B contains 9 red and 5 white marbles. A marble is chosen at random from Box A and its color is recorded. That marble is then placed in Box B and a marble is chosen at random from Box B and its color is recorded.

- Find the probability that a red marble is drawn second.
Box A contains 8 red and 10 white marbles. Box B contains 9 red and 5 white marbles. A marble is chosen at random from Box A and its color is recorded. That marble is then placed in Box B and a marble is chosen at random from Box B and its color is recorded.

- Find the probability that a red marble is drawn second.

\[ P(R_2) = P(RR) + P(WR) \]
\[ = \frac{8 \cdot 10}{18 \cdot 15} + \frac{10 \cdot 9}{18 \cdot 15} \]
\[ = \frac{17}{27} \]
Box A contains 8 red and 10 white marbles. Box B contains 9 red and 5 white marbles. A marble is chosen at random from Box A and its color is recorded. That marble is then placed in Box B and a marble is chosen at random from Box B and its color is recorded.

- Find the probability that you draw a white marble second, given the first marble drawn was white.
Box A contains 8 red and 10 white marbles. Box B contains 9 red and 5 white marbles. A marble is chosen at random from Box A and its color is recorded. That marble is then placed in Box B and a marble is chosen at random from Box B and its color is recorded.

- Find the probability that you draw a white marble second, given the first marble drawn was white.

\[ \frac{6}{15} \]
Box A contains 8 red and 10 white marbles. Box B contains 9 red and 5 white marbles. A marble is chosen at random from Box A and its color is recorded. That marble is then placed in Box B and a marble is chosen at random from Box B and its color is recorded.

• Find the probability that you draw a white marble first, given the second marble drawn was red.
Box A contains 8 red and 10 white marbles. Box B contains 9 red and 5 white marbles. A marble is chosen at random from Box A and its color is recorded. That marble is then placed in Box B and a marble is chosen at random from Box B and its color is recorded.

- Find the probability that you draw a white marble first, given the second marble drawn was red.

\[
P(W_1|R_2) = \frac{P(W_1 \cap R_2)}{P(R_2)}
\]

\[
= \frac{10 \cdot 9}{18 \cdot 15} = \frac{9}{17}
\]
Dice

• If you roll 4 six sided dice, find the probability that the sum of the dice is greater than 5.

• If you roll 4 six sided dice, find the probability that at least one of the dice has a two showing.
Dice

• If you roll 4 six sided dice, find the probability that the sum of the dice is greater than 5.
Use the compliment: For the sum to be 5 or less, we need to roll all four 1s or roll three 1s and one 2. We can do this 5 ways. The total number of outcomes would be $6^4 = 1296$
So $1 - \frac{5}{1296} = .9961$

• If you roll 4 six sided dice, find the probability that at least one of the dice has a two showing.
Dice

- If you roll 4 six sided dice, find the probability that the sum of the dice is greater than 5.

\[
1 - \frac{5}{1296} = 0.9961
\]

- If you roll 4 six sided dice, find the probability that at least one of the dice has a two showing. Use the compliment: For no twos to be showing, each dice has 5 possible outcomes. \(5^4 = 625\)

\[
1 - \frac{625}{1296} = 0.5177
\]