MAT 116 Formula Sheet

Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Midpoint Formula

The midpoint of the line segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Equation of a Circle

The equation for a circle with center (h, k) and radius r with $r > 0$ is $(x - h)^2 + (y - k)^2 = r^2$.

Slope of a Line

The slope of the line through (x_1, y_1) and (x_2, y_2) with $x_1 \neq x_2$ is $\frac{y_2 - y_1}{x_2 - x_1}$.

Equation of a Line

Point-Slope Form: $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

Slope-Intercept Form: $y = mx + b$, where m is the slope and $(0, b)$ is the y-intercept.

Quadratic Formula

The solutions to $ax^2 + bx + c = 0$, with $a \neq 0$, are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Vertex of a Parabola

For a quadratic function of the form $f(x) = ax^2 + bx + c$, the vertex of the parabola is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Average Rate of Change of a Function

If $(x_1, f(x_1))$ and $(x_2, f(x_2))$ are two ordered pairs of a function f, then the average rate of change of f as x varies from x_1 to x_2 is $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$.

Composition of Functions

For two functions f and g, the composition of f and g is defined by $(f \circ g)(x) = f(g(x))$, provided $g(x)$ is in the domain of f.

Transformations

Vertical Translations:

Let $c > 0$.

$y = f(x) + c$ shifts $y = f(x)$ up c units.

$y = f(x) - c$ shifts $y = f(x)$ down c units.

Horizontal Translations:

Let $c > 0$.

$y = f(x - c)$ shifts $y = f(x)$ right c units.

$y = f(x + c)$ shifts $y = f(x)$ left c units.

Reflections:

$y = -f(x)$ reflects $y = f(x)$ across the x-axis.

Stretching:

Let $a > 1$.

$y = af(x)$ stretches $y = f(x)$ by a factor of a.

Shrinking:

Let $0 < a < 1$.

$y = af(x)$ shrinks $y = f(x)$ by a factor of a.

Exponential and Logarithmic Functions

- For $a > 0$, $a \neq 1$, $f(x) = a^x$ and $f(x) = \log_a x$ are inverse functions.
  (Note: $y = \log_a x$ if and only if $x = a^y$.)
- $y = \log x$ if and only if $x = 10^y$.
- $y = \ln x$ if and only if $x = e^y$.

Properties of Exponents

For $a > 0$, m and n positive integers,

$$a^0 = 1 \qquad a^{-m} = \frac{1}{a^m} \qquad a^{m/n} = \sqrt[n]{a^m}$$

Properties of Logarithmic Functions

For $a, b, M, N > 0$ ($a, b \neq 1$) and x a real number,

- $\log_a(a^x) = x$
- $\log_a(MN) = \log_a M + \log_a N$
- $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$
- $\log_a M^x = x \log_a M$
- $\log_a M = \frac{\log_b M}{\log_b a}$