Completing the Square

When **completing the square**, a Perfect Square Trinomial is the goal. These trinomials factor into 2 identical binomial factors. An example would be $x^2 + 6x + 9$ which factors into $(x + 3)(x + 3)$ or $(x + 3)^2$. To know that $x^2 + bx + c$ is a Perfect Square Trinomial, check to see if $c = \left(\frac{b}{2}\right)^2$, which is the square of half the coefficient of *x*. If it is a Perfect Square Trinomial, then it factors as shown below:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2 \text{ or } \left(x + \tfrac{b}{2}\right)\left(x + \tfrac{b}{2}\right)$$

When solving quadratic equations by completing the square, you must force the trinomial to become a Perfect Square Trinomial by adding $\left(\frac{b}{2}\right)^2$. This must be done to *both sides* of the equation to maintain equality.

Completing the Square: Leading Coefficient is 1

Let's solve the equation $x^2 - 6x + 2 = 0$ by completing the square.

$x^2 + 6x + 2 = 0$	*Original Equation is not currently a Perfect Square Trinomial (PST).*
$x^2 + 6x = -2$	*Subtract 2 from both sides to allow a PST to be made.*
$x^2 + 6x + (3)^2 = -2 + (3)^2$ ↓ ↗ $(b/2)^2$	*Divide the 6 by 2, and square it. This is the value needed to have a PST. Add it to both sides to keep the equation balanced.*
$x^2 + 6x + 9 = 7$	*Simplify.*
$(x+3)^2 = 7$	*Factor the Perfect Square Trinomial.*
$x + 3 = \pm\sqrt{7}$	*Solve the equation using square roots.*
$x = -3 \pm \sqrt{7}$	*Solutions*

Completing the Square

Completing the Square: Leading Coefficient is Not 1

Solve the equation $3x^2 - 4x - 5 = 0$ by completing the square.

If the leading coefficient of a quadratic equation is not 1, you should divide both sides of the equation by this coefficient *before* completing the square.

$3x^2 - 4x - 5 = 0$	*Original equation*
$3x^2 - 4x = 5$	*Add 5 to both sides.*
$x^2 - \frac{4}{3}x = \frac{5}{3}$	*Divide both sides by 3. Be sure to divide ALL TERMS.*
$x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 = \frac{5}{3} + \left(-\frac{2}{3}\right)^2$ $\downarrow \quad \nearrow$ $(b/2)^2$	*Divide $-\frac{4}{3}$ by 2, square it, and then add to both sides.* *This is the value needed to have a PST.*
$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{5}{3} + \frac{4}{9}$	*Simplify.*
$\left(x - \frac{2}{3}\right)^2 = \frac{19}{9}$	*Factor the Perfect Square Trinomial.*
$x - \frac{2}{3} = \pm\frac{\sqrt{19}}{3}$	*Solve the equation using square roots.*
$x = \frac{2}{3} \pm \frac{\sqrt{19}}{3}$	*Solutions*

Using a graphing calculator, you can see that the two solutions are approximately 2.11963 and –0.78630, which agree with the two graphical solutions shown below.