

# Domain & Range

## A. The Domain of a Function

"The Domain of a function is the set of all the allowable x or input values."

By using the word "allowable" one could assume there are some values which are simply not allowed into the function.

## B. Examples of RESTRICTIONS on the Domain

### 1. WE DO NOT ALLOW ANY VALUES OF X WHICH WOULD CAUSE US TO DIVIDE BY ZERO.

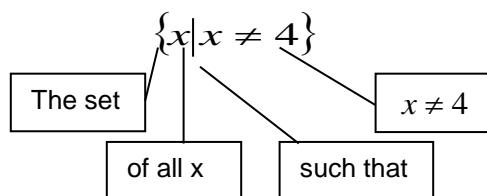
Sometimes we can determine the domain by inspection (just by looking at it).

Example: Find the domain of the following function:

$$f(x) = \frac{3x}{x-4}$$

Can you see that if we let  $x = 4$  the denominator will be zero?

So the domain is:



\*If you are using interval notation, it would look like:  
 $(-\infty, 4) \cup (4, \infty)$

Sometimes it is too difficult to determine the domain by inspection. We set the denominator equal to zero and solve, in order to find the values of  $x$  which would make the denominator zero. We RESTRICT THESE VALUES OF  $X$  FROM THE DOMAIN of the function.

**Example:** Find the domain of the following function:

$$f(x) = \frac{3x}{x^2 - 4x - 12}$$

We can't let the denominator equal zero, so we will SET IT EQUAL TO ZERO TO FIND THE VALUES OF  $X$  WE WILL EXCLUDE FROM THE DOMAIN.

$$x^2 - 4x - 12 = 0$$

$$(x - 6)(x + 2) = 0$$

$$x - 6 = 0 \quad \text{and} \quad x + 2 = 0$$

$$x = 6 \qquad \qquad x = -2$$

These values of  $x$  would make us divide by zero, so we don't let them into the domain.

$$\text{Domain} = \{x | x \neq 6, x \neq -2\}$$

\*If you are using interval notation, it would look like:  
 $(-\infty, -2) \cup (-2, 6) \cup (6, \infty)$

# Domain & Range

2. The second restriction involves taking the even root (like square root) of a negative number. WE DO NOT ALLOW ANY VALUE OF X WHICH WOULD CAUSE US TO TAKE THE SQUARE ROOT OF A NEGATIVE NUMBER.

**Example:** Find the domain of the following:

$$f(x) = \sqrt{6 - 3x}$$

Since we are not allowed to take the square root of a negative number,  $6 - 3x$  **CANNOT BE NEGATIVE**. What is the implication of that? Since the  $6 - 3x$  cannot be negative, then **IT CAN BE POSITIVE OR ZERO**. Thus,

$$6 - 3x \geq 0$$

Solve this inequality to find the values of  $x$  that you **CAN** have in the domain.

$$6 - 3x \geq 0$$

$$-3x \geq -6$$

$$x \leq 2$$

Therefore, the Domain =  $\{x | x \leq 2\}$  or, in interval notation that would be  $(-\infty, 2]$ .

**VERY IMPORTANT:** Notice that our two restrictions involve division and radicals. If your particular problem has other operations, the values for the domain must be considered for that particular function.

## C. The Range of a Function

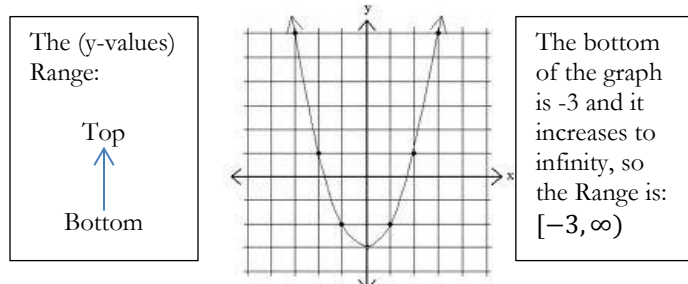
"The Range of the function is the set of resulting  $y$  or output values."

Example: Find the range of the following function:

$$f(x) = x^2 - 3$$

First, note that the domain is the set of all real numbers. We'll let any real numbers into this function. We don't need to worry about the denominator becoming zero, since there is no division. We also don't need to worry about taking the square root (or even root) of a negative number since this function does not involve radicals. There are no restrictions on the  $x$  or input values.

Let's examine the graph of the function to determine the range:



The domain is represented by the  $x$ -values: Left to Right on the graph since the  $x$ -axis runs left to right. This graph keeps moving outward in both directions:  $D = (-\infty, \infty)$