

The **Properties of Exponents** can be used to simplify exponential expressions.

<b>Properties of Exponents</b>	<b>For all nonzero real numbers <math>x</math> and <math>y</math> and integers <math>m</math> and <math>n</math>.</b>	<b>Algebra</b>	<b>Numbers</b>
<b>Product of Powers Property</b>	To multiply powers with the same base, add the exponents.	$x^m \cdot x^n = x^{m+n}$	$4^3 \cdot 4^2 = 4^{3+2} = 4^5$
<b>Quotient of Powers Property</b>	To divide powers with the same base, subtract the exponents.	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{3^7}{3^2} = 3^{7-2} = 3^5$
<b>Power of a Power Property</b>	To raise one power to another, multiply the exponents.	$(x^m)^n = x^{m \cdot n}$	$(4^3)^2 = 4^{3 \cdot 2} = 4^6$
<b>Power of a Product Property</b>	To find the power of a product, apply the exponent to each factor.	$(xy)^m = x^m y^m$	$(3 \cdot 4)^2 = 3^2 \cdot 4^2$
<b>Power of a Quotient Property</b>	To find the power of a quotient, apply the exponent to the numerator and denominator.	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$
<b>Negative Exponent Property</b>	A nonzero base raised to the negative exponent is equal to the reciprocal of the base raised to the positive exponent.	$x^{-n} = \left(\frac{1}{x}\right)^n$ $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$	$7^{-2} = \left(\frac{1}{7}\right)^2$ $\left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4$
<b>Identity Exponent Property</b>	A nonzero quantity raised to the first power is equal to itself.	$x^1 = x$	$8^1 = 8$
<b>Zero Exponent Property</b>	A nonzero quantity raised to the zero power is equal to 1.	$x^0 = 1$	$125^0 = 1$

# Laws of Exponents

For all nonzero real numbers  $x$  and  $y$  and integers  $m$  and  $n$

Product of Powers	$x^m \cdot x^n = x^{m+n}$
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Quotient of Powers	$\frac{x^m}{x^n} = x^{m-n}$
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Power of a Power	$(x^m)^n = x^{m \cdot n}$
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Power of a Product	$(xy)^m = x^m y^m$
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Power of a Quotient	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
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Negative Exponent	$x^{-n} = \left(\frac{1}{x}\right)^n$
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	$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$
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Identity Exponent	$x^1 = x$
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Zero Exponent	$x^0 = 1$
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Summary of Logarithmic Properties (for all formulas, $x$ , $y$ , and $b$ , are all $> 0$ , $a$ and $b \neq 1$ )		
<b>Product Rule:</b>	$\log_b(xy) = \log_b x + \log_b y$	Remember, if you multiply <b>inside</b> the parentheses, you <b>add</b> logs on the outside.
<b>Division Rule:</b>	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	Remember, if you divide <b>inside</b> the parentheses, you <b>subtract</b> logs on the outside.
<b>Power Rule:</b>	$\log_b(x^p) = p \log_b x$	Remember, if you raise to a power <b>inside</b> the parentheses, you <b>multiply</b> by that power on the outside.
<b>Change of Base Rule:</b>	$\log_b(x) = \frac{\log_{10} x}{\log_{10} b}$ or $\log_b(x) = \frac{\ln x}{\ln b}$	Remember, put the larger number (argument) on top, the smaller (base) on bottom. This is useful for the calculator if you don't have the <b>LOGBASE</b> function.

Note that  $\log x$  and  $\ln x$  are not defined when  $x$  is negative or 0.

$$1. \log(AB) = \log A + \log B$$

$$2. \log\left(\frac{A}{B}\right) = \log A - \log B$$

$$3. \log(A^p) = p \log A$$

$$4. \log(10^x) = x$$

$$5. 10^{\log x} = x$$

$$1. \ln(AB) = \ln A + \ln B$$

$$2. \ln\left(\frac{A}{B}\right) = \ln A - \ln B$$

$$3. \ln(A^p) = p \ln A$$

$$4. \ln e^x = x$$

$$5. e^{\ln x} = x$$

In addition,  $\log 1 = 0$  because  $10^0 = 1$ , and  $\ln 1 = 0$  because  $e^0 = 1$ .

Operation	Laws of exponents	Laws of logs
Multiplication	$x^m \cdot x^n = x^{m+n}$	$\log(a \cdot b) = \log(a) + \log(b)$
Division	$\frac{x^m}{x^n} = x^{m-n}$	$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
Exponentiation	$(x^m)^n = x^{mn}$	$\log(a^n) = n \cdot \log(a)$ <i>One of the most useful properties of logs</i>
Zero property	$x^0 = 1$	$\log(1) = 0$
Inverse	$x^{-1} = \frac{1}{x}$	$\log(x^{-1}) = \log\left(\frac{1}{x}\right) = -\log(x)$