Factoring Quadratics

The approaches used in factoring expressions depend on the number of terms that the expression contains.

Remember that your factoring can always be checked by multiplying it out.

2 Terms	**3 Terms**
1. Factor out GCF*	**1. Factor out GCF***
2. Difference of Squares: $a^2 - b^2$	**2. Trinomial with a leading coefficient of 1:** $x^2 + bx + c$
	3. Trinomial with a leading coefficient other than 1: $ax^2 + bx + c$

***No matter how many terms an expression has, factoring out the GCF should always be done FIRST .**

<u>Factoring a Difference of Squares:</u>

Both terms must be perfect squares, and they must be separated by subtraction. If so,

$$a^2 - b^2 \text{ factors into } (a - b)(a + b)$$

Examples: $x^2 - 16 = (x - 4)(x + 4)$

$9x^2 - 25 = (3x - 5)(3x + 5)$

<u>Factoring Quadratic Trinomials with Leading Coefficient of 1:</u>

$x^2 + bx + c$ factors into $(x + p)(x + q)$ by finding the values of p and q that meet the following criteria:

$$p \cdot q = c \quad \boldsymbol{AND} \quad p + q = b$$

Finding p and q:

1. List all possible pairs of factors of c. Remember to include + / - .
2. Determine which factors will add together to give the middle coefficient, b.
   Note: If no factors can be found, it does not factor with this method.

Example: $x^2 - 12x + 27$

Step 1) Factors of c.

1, 27   -1, -27
3, 9   -3, -9

Step 2) Sum of factors equals middle coefficient, b.

$1 + 27 = 28$ $-1 + (-27) = -28$
$3 + 9 = 12$ $-3 + (-9) = -12$ ☺

Now, you can write the factored form $(x + p)(x + q)$ by placing the correct factors p and q.

$(x - 3)(x - 9)$

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Remember to check your answer by multiplying to compare.

$(x - 3)(x - 9)$

$x^2 - 3x - 9x + 27$

$x^2 - 12x + 27$ original: $x^2 - 12x + 27$ ✓

Factoring Quadratic Trinomials with Leading Coefficient Other Than 1:

1. Multiply the leading coefficient and the constant together, $a \cdot c$.
2. List all possible factors of the result from step one.
3. Determine which factors, p and q, will add together to give the middle coefficient, b.
   Note: If no factors can be found, a different form of factoring must be used.
4. Write as $(x + p)(x + q)$.
5. Since we had to multiply by a in step 1, we now need to undo that by dividing p and q by a.
6. If a does not divide into p and q evenly, clear the fraction in that factor.
7. This will give your factored form.
8. Check your answer by multiplying to compare to the original trinomial.

Example: $2x^2 + 17x + 26$

Step 1) Multiply a & c.

$2 \cdot 26 = 52$

Step 2) Factors of $a \cdot c$.

1, 52	-1, -52
2, 26	-2, -26
4, 13	-4, -13

Step 3) Sum of the factors equals middle term.

$1 + 52 = 53$	$-1 - 52 = -53$
$2 + 26 = 28$	$-2 - 26 = -28$
$4 + 13 = 17$ ☺	$-4 - 13 = -17$

Step 4) Write as $(x + p) (x + q)$.

$(x + 4)(x + 13)$

Step 5) Divide p and q by a.

$(x + 2)(x + \frac{13}{2})$

Step 6) Clear fraction left in step 5.

$(x + 2) [2 (x + \frac{13}{2})] = (x + 2) (2x + 13)$

Step 7) Factored form.

$(x + 2)(2x + 13)$

Step 8) Remember to check your answer by multiplying.

$(x + 2)(2x + 13) = 2x^2 + 13x + 4x + 26$ = original: $2x^2 + 17x + 26$ ✓