MAT 112 Formula Sheet

Slope of a Line

The slope of the line through (x_1, y_1) and (x_2, y_2) with $x_1 \neq x_2$ is $\dfrac{y_2 - y_1}{x_2 - x_1}$.

Equation of a Line

Point-Slope Form: $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

Slope-Intercept Form: $y = mx + b$, where m is the slope and $(0, b)$ is the y-intercept.

Quadratic Formula

The solutions to $ax^2 + bx + c = 0$, with $a \neq 0$, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Vertex of a Parabola

- For a quadratic function of the form $f(x) = a(x - h)^2 + k$, the vertex of the parabola is (h, k).
- For a quadratic function of the form $f(x) = ax^2 + bx + c$, the vertex of the parabola is $\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right)$.

Exponential and Logarithmic Functions

- For $a > 0$, $a \neq 1$, $y = \log_a x$ if and only if $x = a^y$.
- $y = \log x$ if and only if $x = 10^y$.
- $y = \ln x$ if and only if $x = e^y$.

Properties of Logarithmic Functions

For $a > 0$, $a \neq 1$, $M, N > 0$ and p a real number,

- $\log_a(MN) = \log_a M + \log_a N$
- $\log_a\left(\dfrac{M}{N}\right) = \log_a M - \log_a N$
- $\log_a M^p = p \log_a M$

Simple Interest

- The simple interest I on P dollars at an annual interest rate r for t years is
$$I = Prt.$$
- The final amount A of P dollars at an annual simple interest rate r for t years is
$$A = P + Prt = P(1 + rt).$$

Compound Interest

- The final amount A of P dollars at an annual interest rate r compounded m times per year for t years is
$$A = P\left(1 + \frac{r}{m}\right)^{mt}.$$
- If a principal is invested at the annual rate r compounded m times a year, then the annual percentage yield is
$$APY = \left(1 + \frac{r}{m}\right)^{m} - 1.$$

Annuities

- For an ordinary annuity of PMT dollars m times a year for t years at an annual interest rate r compounded at the end of each pay period the future value of the annuity is
$$FV = PMT\frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}}.$$
- For an ordinary annuity of PMT dollars m times a year for t years at an annual interest rate r compounded at the end of each pay period the present value is
$$PV = PMT\frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\frac{r}{m}}.$$

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Elementary Row Operations

Any of the following row operations on an augmented matrix gives an equivalent augmented matrix:

- Interchange two rows. $(R_i \leftrightarrow R_j)$
- Multiply a row by a nonzero number. $(aR_i \to R_i)$
- Add a nonzero multiple of one row to another. $(aR_j + R_i \to R_i)$

Principles of Counting

Let A and B be subsets of a universal set U.

- ***Addition Principle:***
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
- ***Complimentary Principle:*** $n(A') = n(U) - n(A)$
- ***Multiplication Principle:*** If k operations are performed in order, with possible number of outcomes $N_1, N_2, \ldots, N_k$, then there are
$$N_1 \cdot N_2 \cdots N_k$$
possible combined outcomes of the operations performed in the given order.

Probability Rules

Let A and B be events in a sample space S.

- ***Addition Rule:***
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
- ***Complimentary Rule:*** $P(A') = 1 - P(A)$
- ***Conditional Probability:*** $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
- ***Product Rule:***
$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$
- ***Independent Events:*** A and B are independent if and only if $P(A \cap B) = P(A)P(B)$

Expected Value of a (Finite) Random Variable X

Given the probability distribution for the (finite) random variable X,

$\boldsymbol{x_i}$	x_1	x_2	$\cdots$	x_n
$\boldsymbol{p_i}$	p_1	p_2	$\cdots$	p_n

where $p_i = P(X = x_i)$, the expected value of X is

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n.$$

Binomial Distributions

If X is the number of successes in n repetitions of a Bernoulli trial with the probability of success of each trial p, then

$$P(X = x) = \text{binompdf}(n, p, x),$$
$$P(X \leq x) = \text{binomcdf}(n, p, x),$$

and the expected value of X is $E(X) = np$.