

# Adding and Subtracting Fractions and Mixed Numbers

A **fraction** is a ratio of two nonzero numbers expressed in the form  $\frac{a}{b}$ .

The top number is called the **numerator** and the bottom number is called the **denominator**.

Fractions are a form of division where the numerator is being divided by the denominator.

$$\frac{1}{2} = 1 \div 2 \quad \text{or} \quad 2 \overline{)1}$$

## A. Types

1. Proper fractions - the denominator is *greater* than the numerator

Proper fractions are numbers like  $\frac{3}{7}$ ,  $\frac{4}{5}$  and  $\frac{11}{16}$ .

2. Improper fractions - the denominator is *smaller* than the numerator

Improper Fractions are numbers like  $\frac{4}{3}$ ,  $\frac{7}{5}$  and  $-\frac{13}{8}$ .

3. Mixed numbers - a whole number with a fraction

Mixed numbers are numbers like  $1\frac{1}{2}$ ,  $3\frac{2}{5}$  and  $-5\frac{3}{8}$ .

A mixed number can be converted into an improper fraction. First, multiply the denominator by the whole number. Then, add the result to the numerator. The denominator will stay the same.

Then, add

Start by multiplying

**Example:**  $2\frac{3}{8} = \frac{(8)(2) + 3}{8} = \frac{16 + 3}{8} = \frac{19}{8}$

To convert an improper fraction to a mixed number, divide the numerator by the denominator. The remainder will be the numerator of the fraction, and the denominator will stay the same.

Ex:  $\frac{22}{7} = 7 \overline{)22} = 7 \overline{)22}^3 = 3\frac{1}{7}$

## B. Reducing Fractions

Fractions are said to be reduced or in lowest terms if the numerator and denominator both cannot be divided by a common factor greater than 1.

**Examples:**  $\frac{4}{10}$  can be reduced or is not in lowest terms because both are divisible by 2.

$\frac{3}{7}$  cannot be reduced since 3 and 7 cannot both be divided by any number other than 1.

To reduce fractions, the numerator and denominator are both divided by a common factor.

**Example:**  $\frac{4}{10}$  can be reduced to  $\frac{2}{5}$  when both 4 and 10 are divided by 2.

## C. Adding and Subtracting Fractions

Fractions MUST have the same denominator to be added or subtracted.

If the denominators are the same, then the numerators are added or subtracted, and the denominator STAYS the same. ALWAYS check to see if the answer can be reduced.

**Examples:**  $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$        $\frac{7}{10} - \frac{3}{10} = \frac{4}{10}$  This can be reduced to  $\frac{2}{5}$ .

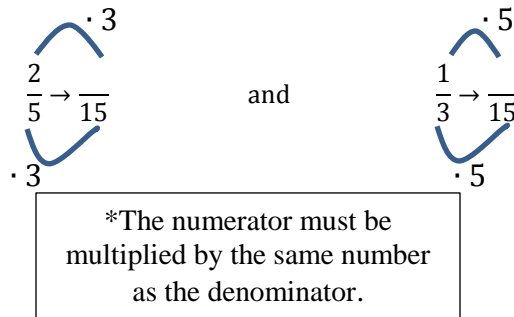
## Adding and Subtracting Fractions and Mixed Numbers

If fractions do not have the same denominator, a common denominator **MUST** be found before the fractions can be added or subtracted.

A common denominator can be found using a variety of methods. In any case, a common multiple is needed for the two different denominator values. This is a number that is divisible by both denominators. It can be found by multiplying the two denominators together. This will not always give you the least common denominator, but it will always work as a common denominator. The final answer just may need to be reduced.

**Example:**  $\frac{2}{5} + \frac{1}{3}$  A common multiple of 3 and 5 is 15. So, this could be used as a common denominator

To get a new denominator for a fraction, the numerator **and** the denominator of the fraction must be multiplied by the same number.



Now, the fractions have a common denominator so they can be added together.

$$\frac{2}{5} + \frac{1}{3} \rightarrow \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$$

### D. Adding and Subtracting Mixed Numbers

To add and subtract mixed numbers, the first step is to convert the mixed numbers to improper fractions. Then, add or subtract them as fractions.

**Example:**  $5\frac{1}{2} - 3\frac{5}{8}$

Convert:  $5\frac{1}{2} = \frac{(5)(2)+1}{2} = \frac{10+1}{2} = \frac{11}{2}$

Convert:  $3\frac{5}{8} = \frac{(3)(8)+5}{8} = \frac{24+5}{8} = \frac{29}{8}$

Rewrite the problem to complete the calculation:  $\frac{11}{2} - \frac{29}{8}$  A common denominator is needed. 8 is a multiple of both. This can be used as a common denominator.

The fraction  $\frac{29}{8}$  has an 8 in the denominator; therefore it does not need to be changed.

$$\frac{11}{2} - \frac{29}{8} \rightarrow \frac{44}{8} - \frac{29}{8} = \frac{15}{8}$$

The answer of  $\frac{15}{8}$  is an improper fraction that we can convert back to the mixed number  $1\frac{7}{8}$ .