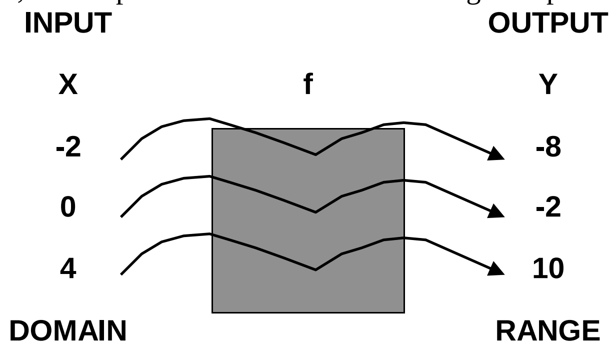
Functions

A. The Concept of a Function

A **function** is a relationship for which each input value is assigned a single output value. In other words, every x-value corresponds to only ONE y-value.

One way to get a better grasp of the concept of a function is to picture the function as a machine. In the machine, each input value is converted to a single output value:



This is a picture of a function with -2, 0, and 4 being put into the function. The set of all *allowable* x (or input) numbers is called the **Domain**. The set of all *resulting* y (or output) numbers is called **Range**. In this example, the domain is {-2, 0, 4} and the Range is {-8, -2, 10}.

There are several ways that functions can be represented. See the figure below:

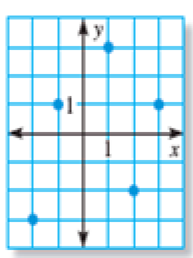
ORDERED PAIRS

{(−2, −3), (−1, 1), (1, 3), (2, −2)}

TABLE

x	−2	−1	0	1	3
y	−4	−4	−4	−4	−4

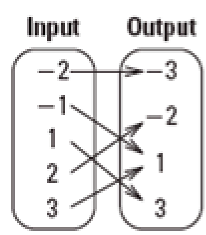
GRAPH



EQUATION

Y = 3X + 1

MAPPING



Examples: Using the definition of a function, is each of the following sets of ordered pairs a function?

1. {(1,2), (3,5), (-2,8)} Yes. Every x goes to only one y.

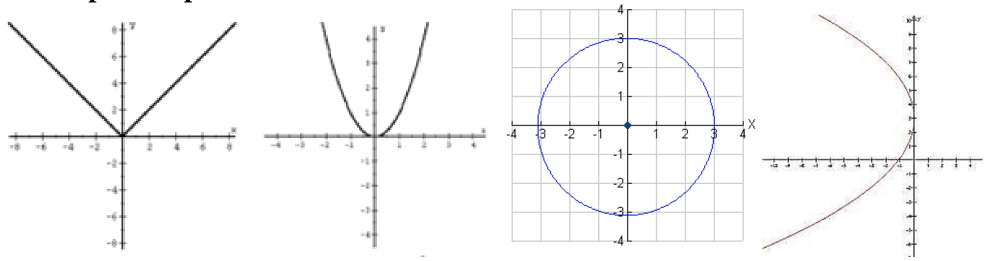
2. {(5,1), (7,1), (10, 1), (-8,2)} Yes. Every x goes to only one y.

3. {(2,6), (3,5), (4,1), (3,7)} No. 3 goes to 5 and 7.

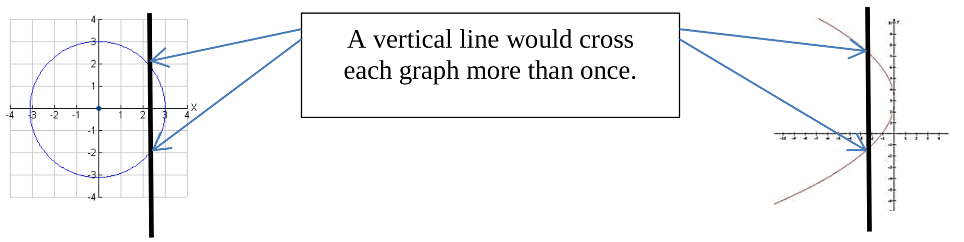
The **Vertical Line Test** can be used to determine if a graph represents a function. If a vertical line can be drawn anywhere through a graph and cross the graph **more than once**, then the graph is **NOT** a function. See the example graphs on the next page:

Functions

Example Graphs:



The first and second graphs are both functions, but the third and fourth are NOT functions.



B. Function Notation

When a function is written using mathematical symbols, the variable *y* becomes f(x), noting that it is a function of x: $y \rightarrow f(x)$

y is a function x

Let's look at both the old way and the new way of notating some values of x and y which satisfy the equation y = 2x + 1:

OLD WAY: y = 2x + 1	FUNCTIONS: f(x) = 2x + 1
Find y when x = 6.	Find f(6).
If x = 0, then y = 1.	f(0) = 1
Find x when y = -5.	Find x if f(x) = -5

Notice a most important fact: *y and f(x) may be used interchangeably.* Also, any letter can be used in function notation: f(x), g(x), h(x), etc.

Example: For $f(x) = x^2 - 2x$, evaluate f(5): *f(5) means that 5 is going into the equation for x

$$\begin{aligned} f(5) &= (5)^2 - 2(5) \\ &= 25 - 10 \\ &= 15 \end{aligned}$$

Example: Let $g(x) = x^2 + 3$. Find g(4)

$$\begin{aligned} g(4) &= 4^2 + 3 \\ &= 16 + 3 \\ &= 19 \end{aligned}$$

*This also means that (4, 19) is a point on the graph of the function.