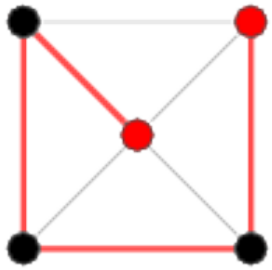
Hamilton Paths and Hamilton Circuits

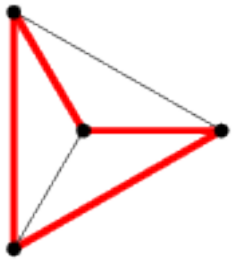
A **Hamilton Path** is a path that goes through every ***Vertex*** of a graph exactly once.

A **Hamilton Circuit** is a **Hamilton Path** that begins and ends at the same vertex.

Hamilton Path



Hamilton Circuit



**notice that not all edges need to be used*

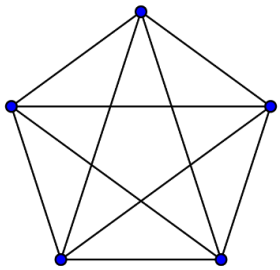
*Unlike Euler Paths and Circuits, there is no trick to tell if a graph has a Hamilton Path or Circuit.

A Complete Graph is a graph where every pair of vertices is joined by an edge.

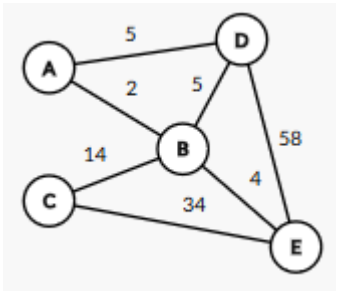
The number of Hamilton circuits in a complete graph with *n* vertices, including reversals, is equal to $(\boldsymbol{n-1})!$ If reversals are not included, the number of Hamilton circuits becomes $\frac{(\boldsymbol{n-1})!}{\boldsymbol{2}}$

A **Weighted Graph** is a graph that has numbers (weights) assigned to its edges.

Complete Graph



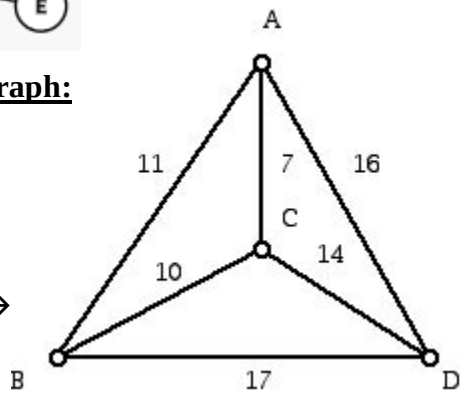
Weighted Graph



Finding a Hamilton Circuit of least weight in a complete weighted graph:

1. **Brute Force Algorithm:**
   a. List all possible Hamilton Circuits.
   b. Find the total weight of each circuit.
   c. The circuit with the smallest weight is the solution.

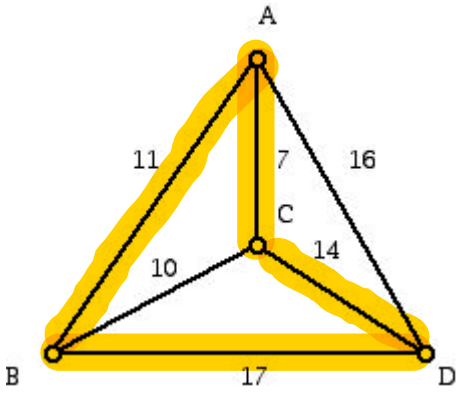
(ABDCA has the smallest weight)→



2. **Nearest Neighbor Algorithm:**
    a. Start at the vertex given.
    b. Choose the edge connected to your starting vertex that has the smallest weight, traveling to a different vertex each time.
    c. Choose new vertices as you did in the previous step making sure not to choose the same vertex more than once.
    d. After all vertices have been chosen, close the circuit by returning to the starting vertex.
        i. Do not close the circuit until all vertices have been chosen

Starting with vertex C

Circuit CABDC, weight 49



3. **Repetitive Nearest Neighbor Algorithm:**
    a. Apply the Nearest Neighbor Algorithm beginning with each vertex and choose the circuit with the smallest weight.

Circuit ACBDA = 50 Circuit BCADB = 50 Circuit DCABD = 49

4. **Best Edge Algorithm:**
    a. Choose any edge with the smallest weight.
    b. Choose any remaining edge with the next smallest weight.
    c. Continue adding the next smallest weight edge while following these conditions:
        i. Do not form a circuit until all vertices have been added
        ii. Do not add an edge that gives a vertex degree 3 or higher

AC - 7
CB - 10
AD - 16
BD - 17

Circuit ACBDA, weight 50