LOGARITHMS

What is a logarithm?

As with anything in mathematics, for one operation, there is an inverse operation for it. The logarithm function ($y = \log_a x$) is the inverse of the exponential function.

Basic Elements of a Logarithm

Let's establish the basics of the logarithm function.

$\log_{10}1000$ is an example of a logarithmic expression

The lowered/subscripted number is the <u>base</u> of the logarithm. Many times the base is not written, and in such cases, 10 is the base for a logarithm. This is called the *common logarithm*.

$\log_{10}1000$ is an equivalent statement to log 1000

To properly refer to this expression, we say: "log base 10 of 1000"

The 1000 is actually the answer when raising the base to an exponent, as you see below:

To evaluate **$\log_{10}1000$**, think : "What exponent do I raise the base 10 to, to get 1000?"

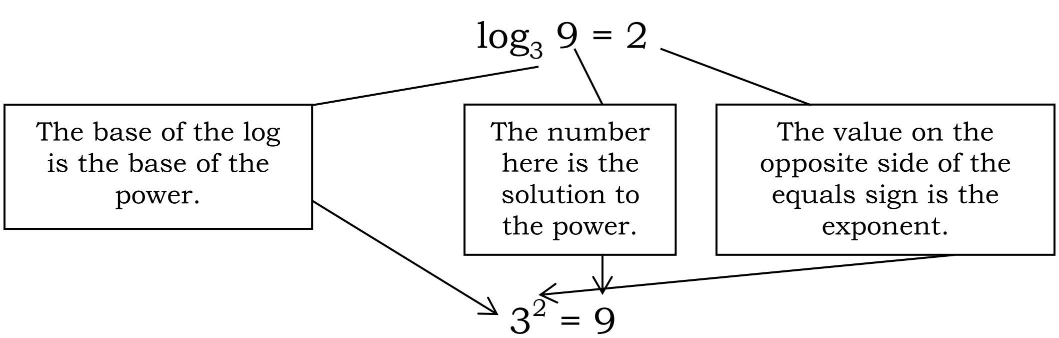
Or, "What exponent satisfies $10^x = 1000$?"

The answer is 3, so $\log_{10}1000 = 3$.

Note: For $\log_a x$, we do not use a = 1 as a base. Also a > 0 and x > 0

Changing Forms (This skill will be used in solving logarithmic and exponential equations.)

In logarithmic form, $\log_a b = x$ is equivalent to the exponential form $a^x = b$.



Properties of Logarithms

1. $\log_b(m \cdot n) = \log_b m + \log_b n$

   Example: $\log_{10}(5 \cdot x) = \log_{10} 5 + \log_{10} x = 0.698970004 + \log_{10} x$

2. $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

   Example: $\log_{10}\left(\frac{10}{x}\right) = \log_{10} 10 - \log_{10} x = 1 - \log_{10} x$

3. $\log_b(m^n) = n \cdot \log_b m$

   Example: $\log_{10} 10^2 = 2 \cdot \log_{10} 10 = 2 \cdot 1 = 2$

4. $\log_b m = \frac{\log_a m}{\log_a b}$

   Example: $\log_5 8 = \frac{\log 8}{\log 5}$ (Each with a base of ten. It can now be evaluated in a calculator.)

   $= 1.292029674$

5. $\log_b(b^n) = n$

   Example: $\log_{10} 10^2 = 2$

6. $b^{\log_b m} = m$

   Example: $5^{\log_5 2} = 2$

7. If $\log_b M = \log_b N$, then $M = N$.

   Example: $\log_5(2x + 1) = \log_5 7$ so, $2x + 1 = 7$ and now solve for x.

8. $\log_b b = 1$

9. $\log_b 1 = 0$

Final Notes

- A logarithmic expression with base *e* ($\log_e x$) is equivalent to ln x; this is *the natural logarithm*.

  Examples:

  log 100 = 2
  ln 100 = 4.605170186