Matrix Addition, Subtraction & Multiplication

A. **A Matrix** is a rectangular array of numbers, in other words, numbers in rows and columns.

B. **The Order of a Matrix** is its size or dimensions. The order is always given as the number of rows by the number of columns (R x C).

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ This matrix is a 3 x 2 matrix.

rows by columns

$\begin{bmatrix} 1 & 5 & 6 & 7 & 8 \\ -2 & 3 & 5 & 4 & -1 \end{bmatrix}$

2 x 5 matrix

C. **For two matrices to be added or subtracted,** the dimensions must be the <u>same</u>. If they are the same, then the corresponding entries are added or subtracted whichever the operation.

Example:

$$\begin{bmatrix} 2 & 5 \\ 3 & -6 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 7 \\ 2 & 4 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ 5 & -2 \\ 0 & 5 \end{bmatrix}$$

D. **For two matrices to be multiplied**, their dimensions need to be analyzed to determine if it is possible. The number of <u>columns</u> of the first matrix MUST EQUAL the number of <u>rows</u> of the second matrix.

Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 6 & 7 & 8 \\ -2 & 3 & 5 & 4 & -1 \end{bmatrix}$

3 x 2 and 2 x 5

They are the same so we <u>can</u> multiply these two matrices.

The outside numbers tell the dimensions or the order of the resulting matrix.

3 x 2 and 2 x 5

The answer will be a 3 x 5 matrix.

The position of each element (row , column) in the answer is a clue to how to multiply.

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) \end{bmatrix}$$

This entry is in the 1st row and 5th column so it is labeled (1 , 5).

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To do the multiplication of the two matrices, a calculation must be completed with the row and columns as follows:

To obtain each entry in the solution matrix, we will look at the row in the first matrix and the column in the second matrix that correspond to the solution matrix entry. So, for the entry that belongs in the solution matrix in the location (1, 5) we will use the 1^{st} row in the first matrix and the 5^{th} column in the second matrix.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 6 & 7 & 8 \\ -2 & 3 & 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{bmatrix}$$

This calculation is for the entry in the 1^{st} row, 5^{th} column.

We will multiply the first entry in each and the second entry in each, and then we will add those two results together:

$$1 \cdot 8 \quad + \quad 2 \cdot -1$$

$$8 \; + \; -2 \; = \; 6$$

$$\begin{bmatrix} \square & \square & \square & \square & 6 \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{bmatrix}$$

This process must be done for each entry in the solution matrix.

Below are a few more examples. Then, the final matrix after all calculations are completed.

Calculating (2, 3)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 6 & 7 & 8 \\ -2 & 3 & 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} \square & \square & \square & \square & 6 \\ \square & \square & 38 & \square & \square \\ \square & \square & \square & \square & \square \end{bmatrix}$$

$$3 \cdot 6 \; + \; 4 \cdot 5$$

$$18 \; + \; 20 \; = \; 38$$

Calculating (3, 4)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 6 & 7 & 8 \\ -2 & 3 & 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} \square & \square & \square & \square & 6 \\ \square & \square & 38 & \square & \square \\ \square & \square & \square & 59 & \square \end{bmatrix}$$

$$5 \cdot 7 \; + \; 6 \cdot 4$$

$$35 \; + \; 24 \; = 59$$

Calculating (1, 1)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 6 & 7 & 8 \\ -2 & 3 & 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} -3 & \square & \square & \square & 6 \\ \square & \square & 38 & \square & \square \\ \square & \square & \square & 59 & \square \end{bmatrix}$$

$$1 \cdot 1 \; + \; 2 \cdot -2$$

$$1 \; + \; -4 \; = -3$$

The final answer for this matrix multiplication: $\begin{bmatrix} -3 & 11 & 16 & 15 & 6 \\ -5 & 27 & 38 & 37 & 20 \\ -7 & 43 & 60 & 59 & 34 \end{bmatrix}$