

# Multiplication, Division, and Exponents of Fractions

## Multiplication

Recall that in order to get a common denominator for

$$\frac{3}{8} + \frac{1}{4}$$

we must multiply

$$\frac{1}{4} \cdot \frac{2}{2} = \frac{2}{8}$$

to get

$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

You just performed fraction multiplication within fraction addition.

Steps for multiplying proper fractions:

1. Multiply the numerators to produce the new numerator.
2. Multiply the denominators to produce the new denominator.
3. Simplify (reduce) the product (if necessary).

Example 1:

$$\frac{2}{5} \cdot \frac{4}{9} = \frac{2(4)}{5(9)} = \frac{8}{45}$$

The product  $\frac{8}{45}$  is already reduced, so the third step is not necessary in this example.

Example 2:

$$\frac{2}{3} \cdot \frac{1}{6} = \frac{2(1)}{3(6)} = \frac{2}{18}$$

In this example, the product  $\frac{2}{18}$  needs to be reduced since both the numerator and the denominator have a Greatest Common Factor (GCF) of 2. So

$$\frac{2}{18} = \frac{(\cancel{2} \cdot 1)}{(3 \cdot 3 \cdot \cancel{2})} = \frac{1}{9}$$

Since proper fractions are not the only type of fractions, it is important to know how to multiply mixed numbers as well. The steps for multiplying mixed numbers are similar to that of proper fractions:

Steps for multiplying mixed numbers:

1. Rewrite the mixed number as an improper fraction.
2. Multiply the numerators to produce the new numerator.
3. Multiply the denominators to produce the new denominator.
4. Simplify (reduce) the product (if necessary).
5. Rewrite the product as a mixed number (if and only if the product is improper).

Example 3:

$$2\frac{1}{3} \cdot 1\frac{1}{2} = \frac{7}{3} \cdot \frac{3}{2} = \frac{21}{6}$$

Reducing, we get

$$\frac{21}{6} = \frac{(7 \cdot \cancel{3})}{(\cancel{3} \cdot 2)} = \frac{7}{2}$$

Since the product is improper, we must rewrite the improper fraction as a mixed number

$$\frac{7}{2} = 3\frac{1}{2}$$

Another approach to simplifying (reducing) fractions: *if the numerator of a fraction has a common factor with the denominator of the other fraction, divide by that common factor. Then multiply.*

Recall the previous example (Example 3):

$$2\frac{1}{3} \cdot 1\frac{1}{2} = \frac{7}{3} \cdot \frac{3}{2}$$

Note how 3 in the denominator of the first fraction and 3 in the numerator of the second fraction have a GCF of 3. Thus dividing by the GCF yields

$$\frac{7}{1} \cdot \frac{1}{2} = \frac{7}{2}$$

The fraction is already reduced and is ready to be rewritten as a mixed number.

## Division

Dividing fractions is similar to fraction multiplication because the division can be rewritten as multiplication.

Example 4:

For example,

$$\frac{3}{8} \div \frac{2}{3}$$

can be rewritten as multiplication as long as the second fraction is reciprocated (switch the numerator and denominator):

$$\frac{3}{8} \cdot \frac{3}{2} = \frac{9}{16}.$$

Example 5:

Similarly,

$$\frac{1}{4} \div \frac{6}{7} = \frac{1}{4} \cdot \frac{7}{6} = \frac{7}{24}.$$

To divide mixed numbers,

1. Rewrite as improper fractions.
2. Flip (reciprocate) the second fraction.
3. Multiply.
4. Simplify (reduce) the product (if necessary).
5. Rewrite the product as a mixed number (if and only if the product is improper).

Example 6:

$$\begin{aligned} 3\frac{2}{3} \div 2\frac{1}{5} &= \frac{11}{3} \div \frac{11}{5} \\ &= \frac{\cancel{11}}{3} \cdot \frac{5}{\cancel{11}} \\ &= \frac{1}{3} \cdot \frac{5}{1} = \frac{5}{3} = 1\frac{2}{3}. \end{aligned}$$

## Exponents

When a fraction is raised to a power, it is the same as raising both the numerator and the denominator to the power.

Example 7:

$$\left(\frac{2}{5}\right)^2 = \frac{2^2}{5^2} = \frac{4}{25}.$$

Then for fractions raised to a power:

1. Rewrite as an improper fraction (if it is a mixed number)
2. Raise the numerator to the power to get the new numerator.
3. Raise the denominator to the power to get the new denominator.
4. Simplify (reduce) the product (if necessary).
5. Rewrite the product as a mixed number (if and only if the product is improper).

Example 8:

$$\begin{aligned}\left(1\frac{1}{2}\right)^3 &= \left(\frac{3}{2}\right)^3 \\ &= \frac{3^3}{2^3} = \frac{27}{8} = 3\frac{3}{8}.\end{aligned}$$

Remember that order of operations ALWAYS applies, even with fractions. Now try some on your own—these may combine many of the operations previously discussed.

1.  $\frac{6}{7} \cdot \frac{3}{5} =$

5.  $4\frac{3}{8} \div 2\frac{5}{6} =$

2.  $\frac{2}{3} \cdot 9\frac{1}{5} =$

6.  $\left(1\frac{3}{7}\right)^2 \cdot 4\frac{1}{12} =$

3.  $\frac{3}{5} \cdot 2\frac{4}{9} =$

7.  $\left(2\frac{2}{5}\right)^2 \div \left(\frac{2}{3}\right)^3 =$

4.  $5 \div \frac{5}{6} =$

Answer Key:

1)  $\frac{18}{35}$

2)  $6\frac{2}{15}$

3)  $1\frac{7}{15}$

4) 6

5)  $1\frac{37}{68}$

6)  $8\frac{1}{3}$

7)  $19\frac{11}{25}$