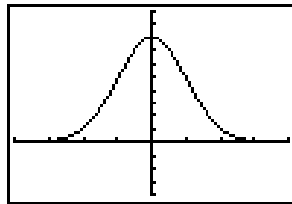# Normal Distribution

Normal distribution is a continuous, symmetric, bell-shaped distribution of a variable.

   μ = mean σ = standard deviation

Summary of Properties

   1) The normal distribution curve is bell-shaped
   2) The mean, median, and mode are equal and located at the center of the distribution
   3) The curve is symmetrical about the mean
   4) The curve never touches the x-axis
   5) The total area under the curve is equal to 1.00
   6) The area under the curve that lies within one standard deviation of the mean is 0.68 or 68%; within two standard deviations is 0.95 or 95%; within three standard deviations is 0.997 or 99.7%

Standard Normal Distribution is a normal distribution with a mean of 0 ($\mu = 0$) and a standard deviation of 1 ($\sigma = 1$).

   - Any normally distributed variable can be transformed into a standard normal distribution by using the formula for the standard score or z-value

   $$z = \frac{x - \mu}{\sigma}$$

   where the z-value is the number of standard deviations that a particular x value is away from the mean.

Example: Find the z-value for a normally distributed variable with a mean of 4, standard deviation of 2, and an x value of 8.

$$z = \frac{8 - 4}{2} = \frac{4}{2} = 2$$

Note: We have found that z = 2, which tells us that the x-value of 8 is 2 standard deviations away from the mean of 4 when the standard deviation is 2.

Application: Using the z-value, we can use the standard normal distribution table to find the area under the curve. The area is used to solve problems such as finding percentages or probabilities.

Finding the Area Under the Standard Normal Distribution Curve

1) Between 0 and any z-value:
      a) Look up the z-value in the table to get the area

Normal Distribution

Example: The average score on Dr. Vladamere's math test was a 76 with a standard deviation of 5. To get an A, a student must have a score of 90 or higher; a B is 80-90; a C is 70-80; and a D is 60-70. What is the probability a randomly selected student scored between a 76 and 90? Assume the variable is normally distributed.

First draw the normal curve. The average score of 76 tells us that we have a mean of 76.
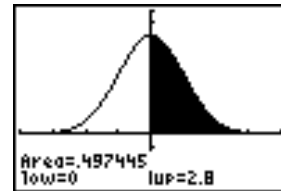
$\mu = 76$, $\sigma = 5$, $x = 90$

Area=.497445
low=76   up=90
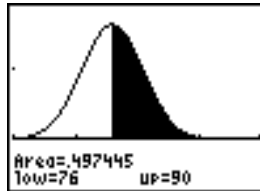
Next find the z-value:

$$z = \frac{x - \mu}{\sigma} = \frac{90 - 76}{5} = 2.8$$

Then draw the standard normal curve with the found z-value:

Area=.497445
low=0   lup=2.8

Lastly, look up the z-value on the table. (A sample table is on the last page of this handout.):
    With a z-value of 2.8, we find the area to be .4974.

Therefore, the probability a randomly selected student scored between 76 and 90 is .4974 or 49.74%.
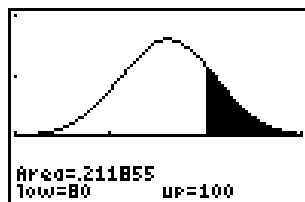
2) In any tail:
    a) Look up the z-value to get the area
    b) Subtract the area from 0.500

Example: Using the same information from the previous example, what is the probability a randomly selected student will have received at least a B?

First draw the normal curve:

$\mu = 76$, $\sigma = 5$, $x = 80$

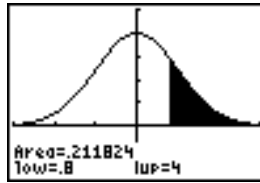Area=.211855
low=80   up=100

Note: To receive at least a B, a student must score an 80 or better.

Next find the z-value:

$$z = \frac{x - \mu}{\sigma} = \frac{80 - 76}{5} = 0.8$$

Normal Distribution

Then draw the standard normal curve:

Area=.211824
low=.8   up=4

Next, look up the z-value on the table. (A sample table is on the last page of this handout.):

A z-value of 0.8 gives an area of 0.2881

Lastly, subtract that area from 0.500.

0.500 − 0.2881 = 0.2119

Therefore, the probability a randomly selected student received at least a B is <u>0.2119</u> or <u>21.19%</u>.

3) Between two z-values on opposite sides of the mean:

a) Look up both z-values

b) Add the areas

Example: Again using the same information, what is the probability a randomly selected student will score between a 71 and 84?

First draw the normal curve:

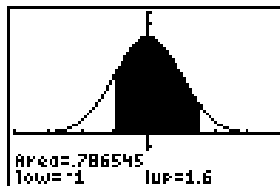$$\mu = 76, \ \sigma = 5, \ x_1 = 71, \ x_2 = 84$$

Area=.786545
low=71   up=84
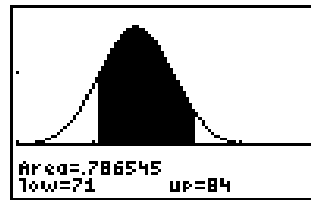
Next find the z-values:

$$z_1 = \frac{71 - 76}{5} = -1 \qquad\qquad z_2 = \frac{84 - 76}{5} = 1.6$$

Then draw the standard normal curve:

Area=.786545
low=-1   up=1.6

Next look up the z-values on the table (A sample table is on the last page of this handout.):

A z-value of -1 gives us and area of 0.3413 and a z-value of 1.6 gives an area of 0.4452

Note: Even though z_1 is negative, we look up 1 on the table.

Lastly, add the areas together:

0.4452 + 0.3413 = 0.7865

Therefore, the probability a randomly selected student scored between 71 and 84 is <u>0.7865</u> or <u>78.65%</u>.
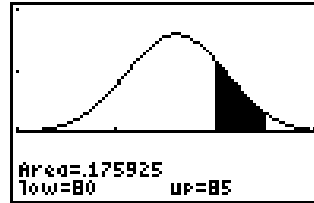
Normal Distribution

4) Between two z-values on the same side of the mean:
　　　a) Look up both z values
　　　b) Subtract the smaller area from the larger

Example: Still using the same information, what is the probability a randomly selected student will score between an 80 and 85?

First draw the normal curve:

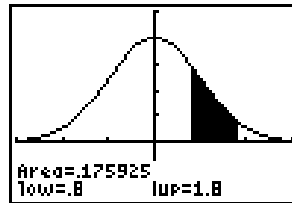$$\mu = 76,\ \sigma = 5,\ x_1 = 80,\ x_2 = 85$$

Next find the z-values:

$$z_1 = \frac{80 - 76}{5} = 0.8 \qquad\qquad z_2 = \frac{85 - 76}{5} = 1.8$$

Then draw the standard normal curve:

Next look up the z-values on the table (A sample table is on the last page of this handout.):
　　　A z-value of 0.8 gives an area of 0.2881 and a value of 1.8 gives 0.4641

Lastly, subtract the smaller area from the larger:
　　　$0.4641 - 0.2881 = .1760$

Therefore, the probability a randomly selected student will score between an 80 and 85 is .1760 or 17.60%.

5) The left of any z-value, where z is greater than the mean:
　　　a) Look up the z-value to get the area
　　　b) Add 0.500 to the area.

Example: Still using the same information, what is the probability a randomly selected student will <u>not</u> receive an A or B?

First draw the normal curve:
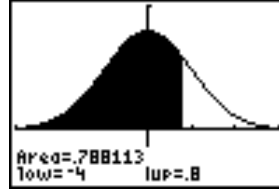
$$\mu = 76,\ \sigma = 5,\ x = 80$$
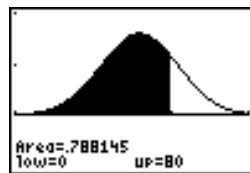
Note: To get an A or B, a student must score an 80 or above. We want the probability of not getting an A or B so look at everything below an 80.

Normal Distribution

Next find the z-value:
$$z = \frac{80 - 76}{5} = 0.8$$

Then draw the standard normal curve:

Next look up the z-value on the table (A sample table is on the last page of this handout.):
   A z-value of 0.8 has an area of 0.2881.

Lastly, add the area to 0.500:
   $0.500 + 0.2881 = 0.7881$

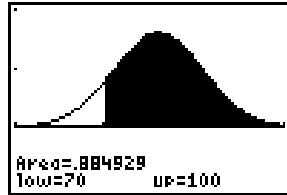Therefore, the probability a randomly selected student did not receive an A or B is <u>0.7881</u> or <u>78.81%</u>.

6) To the right of any z-value, where z is less than the mean:
   a) Look up the z-value to get the area
   b) Add 0.500 to the area

Example: Again using the same information, what is the probability a randomly selected student will score at least a 70?
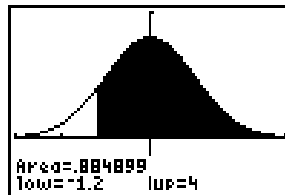
First draw the normal curve:

   μ = 76, σ = 5, x = 70

Next, find the z-value:

$$z = \frac{70 - 76}{5} = -1.2$$

Then, draw the standard normal curve:

Next, look up the z-value (A sample table is on the last page of this handout.):
   A z-value of -1.2 has an area of 0.3849

Lastly, add the area to 0.500:
   $0.500 + 0.3849 = .8849$

Therefore, the probability a randomly selected student will score at least a 70 is <u>0.8849</u> or <u>88.49%</u>.