

# Rationalizing the Denominator

## A. What It Means to Rationalize the Denominator

In order that all of us doing math can compare answers, we agree upon a common conversation, or set of rules, concerning the form of the answers. For instance, we could easily agree that we would not leave an answer in the form of  $3 + 4$  but would write  $7$  instead.

When the topic switches to that of radicals, those doing math have agreed that a **radical in simplest form** will not (among other things) have a radical in the denominator of a fraction. We will all change the form so there is no radical in the denominator.

If a fraction contains a radical in the denominator such as  $\sqrt{3}$  which is an irrational number, we need to make it NOT irrational, or rational. The process of changing its form so it is no longer irrational is called **rationalizing the denominator**.

## B. Different Cases of Rationalizing the Denominator

**Case 1:** A single square root in the denominator.

Example:  $\frac{7}{\sqrt{3}}$

square root

one term

In a fraction, the numerator and denominator can be multiplied by the same value and still be an equivalent fraction. For example,  $\frac{1}{2} = \frac{3}{6}$  for which both the top and bottom of the fraction were multiplied by 3. We will use this idea in this example to create an equivalent fraction, but it will have a rational denominator.

**Procedure:** Multiply the numerator and denominator of the fraction by the radical that is currently in the denominator.

$$\frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{\sqrt{9}} = \frac{7\sqrt{3}}{3}$$

Look at what is happening here: The denominator becomes  $\sqrt{9}$  which is a square root that can be simplified. It simplifies to 3.

Now, the denominator is a rational number which was our goal.

**Case 2:** There is a single radical in the denominator, however, THE INDEX IS GREATER THAN TWO. It might be a cube root or a fourth root.

Example:  $\frac{2}{\sqrt[3]{3x^2}}$

We need to multiply by something that will allow us to completely simplify the cube root. Currently there is one 3 and two x's under the radical:  $\sqrt[3]{3 \cdot x \cdot x}$ . So, we need two more 3's ( $3 \cdot 3 = 9$ ) and another x to be able to completely simplify the cube root.

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*Procedure:* Multiply the numerator and denominator by the radical that when multiplied creates a perfect cube under the radical in the denominator.

$$\frac{2}{\sqrt[3]{3x^2}} \cdot \frac{\sqrt[3]{9x}}{\sqrt[3]{9x}} = \frac{2\sqrt[3]{9x}}{\sqrt[3]{27x^3}}$$

$$= \frac{2\sqrt[3]{9x}}{3x}$$

So we can completely simplify the cube root.

This now leaves a rational denominator.

**Case 3:** There are TWO TERMS in the denominator.

*Example:*  $\frac{2}{3-\sqrt{5}}$

*Procedure:* We will multiply both numerator and denominator by the conjugate of the denominator. The conjugate has the same two terms but with the opposite sign between them.

The conjugate of  $3 - \sqrt{5}$  is  $3 + \sqrt{5}$ . Because of this, the denominator will simplify to a rational number.

$$\frac{2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$$

Be careful with the multiplication in both the numerator and denominator.

$$= \frac{6+2\sqrt{5}}{9+3\sqrt{5}-3\sqrt{5}-\sqrt{25}}$$

Notice that the middle terms cancel.

$$= \frac{6+2\sqrt{5}}{9-\sqrt{25}}$$

$$= \frac{6+2\sqrt{5}}{9-5}$$

$$= \frac{6+2\sqrt{5}}{4}$$

The denominator will now simplify to a rational number.

Lastly, this fraction can be reduced. Since there are two terms in the numerator, we must be able to reduce **both** of them with the denominator. In this case, we can:

$$\frac{\cancel{6}+2\sqrt{5}}{\cancel{4}} = \frac{3+1\sqrt{5}}{2} = \frac{3+\sqrt{5}}{2}$$

This is our final answer.