Solving Exponential & Logarithmic Equations

I. To Solve Exponential Equations (variable in exponent position):

A. When the bases are the same:

Solve: $3^{X+4} = 3^{2X-1}$

STEPS: When the bases are the same, set the exponents equal to each other. Solve for the variable. Always check the solutions by substitution.

$x + 4 = 2x - 1$
$-x = -5$
$x = 5$

B. When the bases are not the same and **NOT** e:

Solve: $3^{X+4} = 5^{X-6}$

STEPS: Take the log of both sides. Move the exponents in front of the log. Use the distribution property and solve for x.

$\log 3^{X+4} = \log 5^{X-6}$
$(x + 4)\log 3 = (x - 6)\log 5$
$x \log 3 + 4 \log 3 = x \log 5 - 6 \log 5$
$x \log 3 - x \log 5 = -4 \log 3 - 6 \log 5$
$x (\log 3 - \log 5) = -1 (4 \log 3 + 6 \log 5)$
$$x = \frac{-1(4\log 3 + 6\log 5)}{\log 3 - \log 5}$$

C. When the base is e:

Solve: $e^{2x-5} = 29$

STEPS: Take the ln (natural log) of both sides. Move the exponents to the front of the ln. Since ln e = 1, 2x – 5 times ln e is 2x – 5. Solve for x.

$\ln e^{2x-5} = \ln 29$
$(2x - 5)\ln e = \ln 29$
$2x - 5 = \ln 29$
$2x = \ln 29 + 5$
$$x = \frac{(\ln 29) + 5}{2}$$

II. To Solve Logarithmic Equations (log or ln):

A. When every term has the word log (or ln):

Solve: $\log(x - 3) + \log x = \log 18$

STEPS: Use properties of logarithms to condense one side to a single log. Both sides of equation have the same base. Therefore, we can cancel the logs, and solve for the indicated variable.

$\log[x(x - 3)] = \log 18$
$x(x - 3) = 18$
$x^2 - 3x - 18 = 0$
$(x - 6)(x + 3) = 0$
$x = 6,\quad \cancel{x = -3}$

Since it is impossible to take the log of a negative number, -3 does not check in the original problem.

B. When not every term has log (or ln):

Solve: $\log(x + 2) - \log x = 2$

STEPS: Use properties of logarithms to condense logs into one term. Change from log form to exponential form.** Solve for the indicated variable.

$\log\left[\dfrac{(x + 2)}{x}\right] = 2$

$10^2 = \dfrac{(x + 2)}{x}$

$100 = \dfrac{(x + 2)}{x}$

$100x = x + 2$
$99x = 2$
$x = \dfrac{2}{99}$

****In logarithmic form, $\log_a b = x$ is equivalent to the exponential form $a^x = b$**