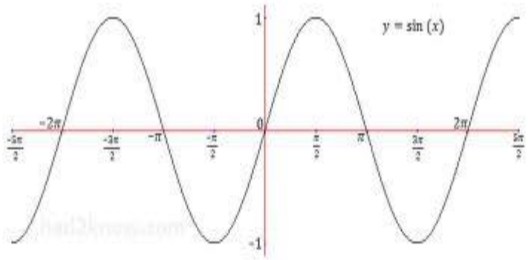
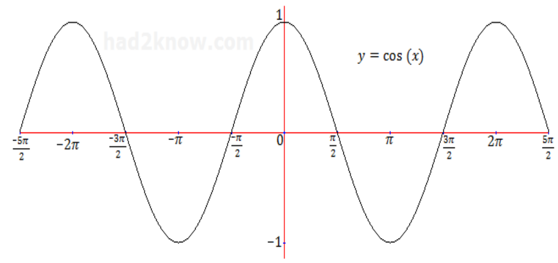
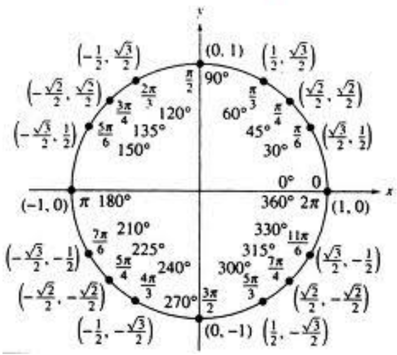
Graphing Sine and Cosine

First, let's look at the parent graphs of sine and cosine:







(x, y) $\longrightarrow$ (cosθ, sinθ)

You can see how the points on the graph of each parent function correlates to the values for that function on the unit circle. For example:

sin(0) = 0 and (0, 0) is a point on y = sin(x)

cos($\frac{\pi}{2}$) = 0 and ($\frac{\pi}{2}$, 0) is a point on y = cos(x)

sin($\frac{\pi}{2}$) = 1 and ($\frac{\pi}{2}$, 1) is a point on y = sin(x)

Notes:

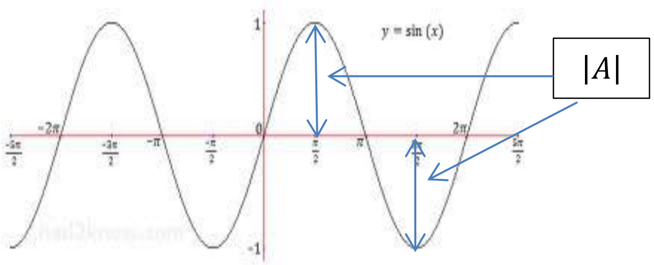
- The sine parent graph crosses through the origin.
- The sine and cosine parent graphs each oscillate between y = -1 and y = 1.
- The ordered pairs for these graphs were derived from the unit circle.

To graph sine and cosine, use the general forms:

$$y = Asin[B(x - C)] + D \qquad\qquad y = Acos[B(x - C)] + D$$

Transformations of the parent graphs can include:

1. A change in ***amplitude***: The amplitude is $|A|$. Graphically it is the distance from the midline to the top and bottom of the graph. The amplitude of the parent graphs is 1.



2. A ***reflection*** over the x-axis: If A < 0, then the graph is reflected over the x-axis.

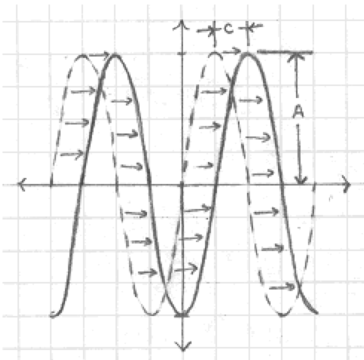
3. A change in the ***period*** of the function: The period of sine and cosine functions is found by evaluating $\frac{2\pi}{B}$ for B > 0. The period of a function is the length of one cycle. The period of the parent graphs of sine and cosine is 2π since B = 1.

Graphing Sine and Cosine

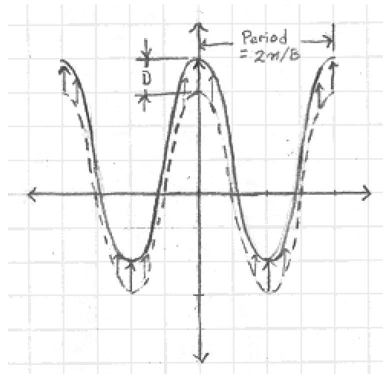
4. ***Horizontal translation*** also called a phase shift: The phase shift of the parent graph is C. This is how far the graph is shifted to the right (for C > 0) or to the left (for C < 0).

5. ***Vertical Translation***: The distance that the graph is shifted vertically is D. The graph is shifted up for D > 0 and down for D < 0.

Example Graphs:



Sine graph shifted C units
to the right with amplitude A.



Cosine graph shifted D units up
with a period of $\frac{2\pi}{B}$.

Example Problem:
Determine the amplitude, period, phase shift, and vertical shift. Then graph two cycles of the function:

$$y = 3\sin(2x + \frac{\pi}{2})$$

1. Rewrite in general form by factoring a 2 out of 2x + $\frac{\pi}{2}$: $y = 3\sin[2\left(x + \frac{\pi}{4}\right)]$

   From this equation we get:
   A = 3, B = 2, C = - $\frac{\pi}{4}$ and D = 0

   So, the amplitude is A = 3, the period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$, the phase shift is $\frac{\pi}{4}$ units to the left, and there is no vertical shift.
2. Find 5 key points on the graph by using the 5 key x-values from the parent graphs: 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2 π.

   a. Divide each x-coordinate from the parent graph by B and then add C.
   b. Multiply each y-coordinate by A and then add D.

5 key points on the parent graph of y =sin(x)	(0, 0)	($\frac{\pi}{2}$, 1)	(π, 0)	($\frac{3\pi}{2}$, -1)	(2π, 0)
5 key points on the transformed graph of $y = 3\sin[2\left(x + \frac{\pi}{4}\right)]$	($-\frac{\pi}{4}$, 0)	(0, 3)	($\frac{\pi}{4}$, 0)	($\frac{\pi}{2}$, -3)	($\frac{3\pi}{4}$, 0)

3. Sketch a graph using these five points: