MAT110 REVIEW

Counting and Probability

COUNTING METHODS

- **Fundamental Counting Principle** is used when we perform a series of tasks. If the first task can be done in *a* ways, the second task can be done in *b* ways, the third task can be done in *c* ways, and so on, then all the tasks can be done in $\mathrm{a} \cdot b \cdot c \cdots$ ways
- **Slot Diagrams** can be used to visualize the number of ways each task can be done before using Fundamental Counting Principle to find a total.
- A **Permutation** is an ordering of distinct objects in a straight line. *Order matters*
  $P(n, r) = \frac{n!}{(n-r)!}$
- If we choose objects from a set, we are forming a **Combination**.
  $C(n, r) = \frac{n!}{r!(n-r)!}$

EXAMPLE

A security keypad uses five digits (0 to 9) in a specific order. How many different keypad patterns are possible if any digit can be used in any position and repetition is allowed?

10	✖	10	✖	10	✖	10	✖	10
Any Digit		Any Digit		Any Digit		Any Digit		Any Digit

- =100,000 possibilities

EXAMPLE

A security keypad uses five digits (0 to 9) in a specific order. How many different keypad patterns are possible if the first digit cannot be 0 and the last two digits must be even?

9	✖	10	✖	10	✖	5	✖	5
Not Zero		Any Digit		Any Digit		Even		Even

- =22,500 possibilities

EXAMPLE

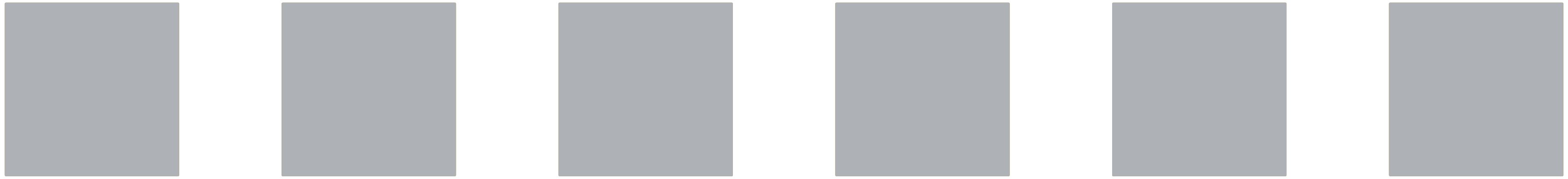
A college class has 15 students. Max must sit in the front row next to his tutor, Griffon. If there are six chairs in the front row of the classroom, how many different ways can students be assigned to sit in the front row?

- First, determine our tasks:
  - Task 1: Assign Max and Griffon a pair of chairs next to one another.
  - Task 2: Arrange Max and Griffon in the pair of chairs.
  - Task 3: Assign students to the remaining front row chairs.

EXAMPLE

A college class has 15 students. Max must sit in the front row next to his tutor, Griffon. If there are six chairs in the front row of the classroom, how many different ways can students be assigned to sit in the front row?

- Task 1: Assign Max and Griffon a pair of chairs next to one another.



- There are **5** ways to assign Max and Griffon a pair of chairs.

EXAMPLE

A college class has 15 students. Max must sit in the front row next to his tutor, Griffon. If there are six chairs in the front row of the classroom, how many different ways can students be assigned to sit in the front row?

- Task 2: Arrange Max and Griffon in the pair of chairs.

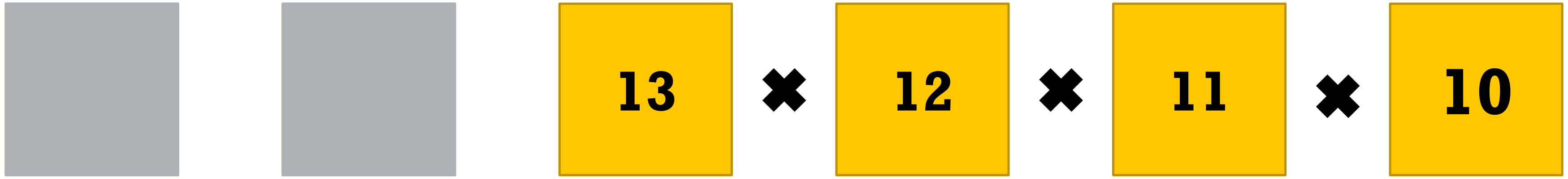
M **G** OR **G** **M**

- There are **2** ways to arrange Max and Griffon.

EXAMPLE

A college class has 15 students. Max must sit in the front row next to his tutor, Griffon. If there are six chairs in the front row of the classroom, how many different ways can students be assigned to sit in the front row?

- Task 3: Assign students to the remaining front row chairs.



- There are **17160** ways to assign Max and Griffon a pair of chairs.
- Note: You could also do $P(13, 4)$

EXAMPLE

A college class has 15 students. Max must sit in the front row next to his tutor, Griffon. If there are six chairs in the front row of the classroom, how many different ways can students be assigned to sit in the front row?

- Use Fundamental Counting Principle to get the final answer:
  - Task 1: Assign Max and Griffon a pair of chairs next to one another. **5**
  - Task 2: Arrange Max and Griffon in the pair of chairs. **2**
  - Task 3: Assign students to the remaining front row chairs. **17160**

- $\mathbf{5 \cdot 2 \cdot 17160 = 171600}$
- There are **171,600** ways to seat Max and Griffon in the front row.

EXAMPLE

- License plates in Florida have the form A24BCDE; that is, a letter followed by 2 digits followed by 4 more letters.
- How many license plates are possible?

26	✖	10	✖	10	✖	26	✖	26	✖	26	✖	26
Letter		Digit		Digit		Letter		Letter		Letter		Letter

- There are **1,188,137,600** possible license plates.

EXAMPLE

- License plates in Florida have the form A24BCDE; that is, a letter followed by 2 digits followed by 4 more letters.
- Griff would like his license plate to end in MWSU. How many plates are possible?

26	✖	10	✖	10	✖	1	✖	1	✖	1	✖	1
Letter		Digit		Digit		M		W		S		U

- There are **2,600** possible license plates.

EXAMPLE

- The math club needs to select a President, Vice President, Treasurer and Secretary from their 28 members. How many ways can they do this?
  - Note: Members are being assigned positions, so this is a permutation.

- $P(28, 4) = 491400$

- There are **491,400** possible leadership committees.

EXAMPLE

- In the game of poker, five cards are drawn from a standard 52-card deck. How many different poker hands are possible?
  - Note: Order does not matter, so this is a combination.

- $C(52, 5) = 2598960$

- There are **2,598,960** possible hands.

EXAMPLE

- A club has 10 seniors, 7 juniors, 8 sophomores, and 5 freshman. How many ways can the club choose a 5 person planning committee with exactly 3 juniors?

- Task 1: Choose the 3 juniors
  - $\mathrm{C}(7,3) = 35$
- Task 2: Choose 2 more that are not juniors
  - $C(23,2) = 253$

- Use Fundamental Counting Principle to get the final answer: $35 \cdot 253 = 8855$
- There are **8,855** ways to create the planning committee.

EXAMPLE

- A 18-member organization wishes to choose a committee consisting of a president, vice president, secretary, and a four-member executive board. In how many different ways can this committee be formed?

- Task 1: Choose the president, vice president, and secretary.
  - $P(18, 3) = 4896$
- Task 2: Choose the 4-member executive committee from the remaining members.
  - $C(15, 4) =$1365

- Use Fundamental Counting Principle to get the final answer: $4896 \cdot 1365 = 6683040$
- There are **6,683,040** ways to create the committee.

PROBABILITY DEFINITIONS

- In an experiment, the **sample space** is a list of all possible outcomes.
- An **event** is a subset of the sample space.
- The **probability of event E**, written $P(E)$, is the sum of the probabilities of the outcomes in event E.

Let E and F be events in a sample space S. Then

1. $0 \leq P(E) \leq 1$ 2. $P(S) = 1$ 3. $P(\emptyset) = 0$

4. $P(\bar{E}) = 1 - P(E)$

5. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

EQUALLY LIKELY OUTCOMES

A couple wants three children. What are the arrangements of boys (B) and girls (G)?

- Find the probability of an individual outcome in your sample space. Genetics tells us that the probability that a baby is a boy or a girl is the same, 0.5.
  - Sample space: {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}
  - All eight outcomes in the sample space are equally likely. The probability of each is thus 1/8.

MARBLE EXAMPLE

A jar contains 4 red marbles, 11 green marbles, and 6 blue marbles.

- What is the probability that you draw 4 green marbles in a row if you *do replace* the marbles after each draw?

$\frac{11}{21}$	✖	$\frac{11}{21}$	✖	$\frac{11}{21}$	✖	$\frac{11}{21}$
First Marble		Second Marble		Third Marble		Forth Marble

- The probability of drawing 4 green marbles is $\frac{14641}{194481} = 0.07528$

MARBLE EXAMPLE

A jar contains 4 red marbles, 11 green marbles, and 6 blue marbles.

- What is the probability that you draw 3 red marbles in a row if you *don't replace* the marbles after each draw?

$\frac{4}{21}$	✖	$\frac{3}{20}$	✖	$\frac{2}{19}$
First Marble		Second Marble		Third Marble

- The probability of drawing 3 red marbles is $\frac{24}{7980} = 0.003008$

MARBLE EXAMPLE

A jar contains 4 red marbles, 11 green marbles, and 6 blue marbles.

- What is the probability that you draw 7 blue marbles in a row if you *don't replace* the marbles after each draw?

- Note: There are only 6 blue marbles in the jar. It is impossible to draw 7 blue marbles without replacement.

- The probability of drawing 7 blue marbles is **0**.

MARBLE EXAMPLE

A jar contains 4 red marbles, 11 green marbles, and 6 blue marbles.

- What is the probability that you draw exactly one of each color, if you pick three from the jar?

$\frac{4}{21}$	✖	$\frac{11}{20}$	✖	$\frac{6}{19}$	✖	$3!$
Red Marble		Green Marble		Blue Marble		Ways to order three colors

- The probability of drawing one of color is $\frac{1584}{7980} = 0.198496$

EXAMPLE

- Three students from an 18 student class will be selected to attend a meeting. 11 of the students are female.
  - What is the probability that all three of the students chosen to attend the meeting are female?

$$\frac{Group\ of\ 3\ female\ students}{Group\ of\ any\ 3\ students} = \frac{C(11,3)}{C(18,3)} = \frac{165}{816}$$

The probability of selecting all three females is $\mathbf{\frac{165}{816} = 0.2022}$

EXAMPLE

- Three students from an 18 student class will be selected to attend a meeting. 11 of the students are female.
  - What is the probability that NOT all three of the students chosen to attend the meeting are female?
  - Note: We can use the compliment!

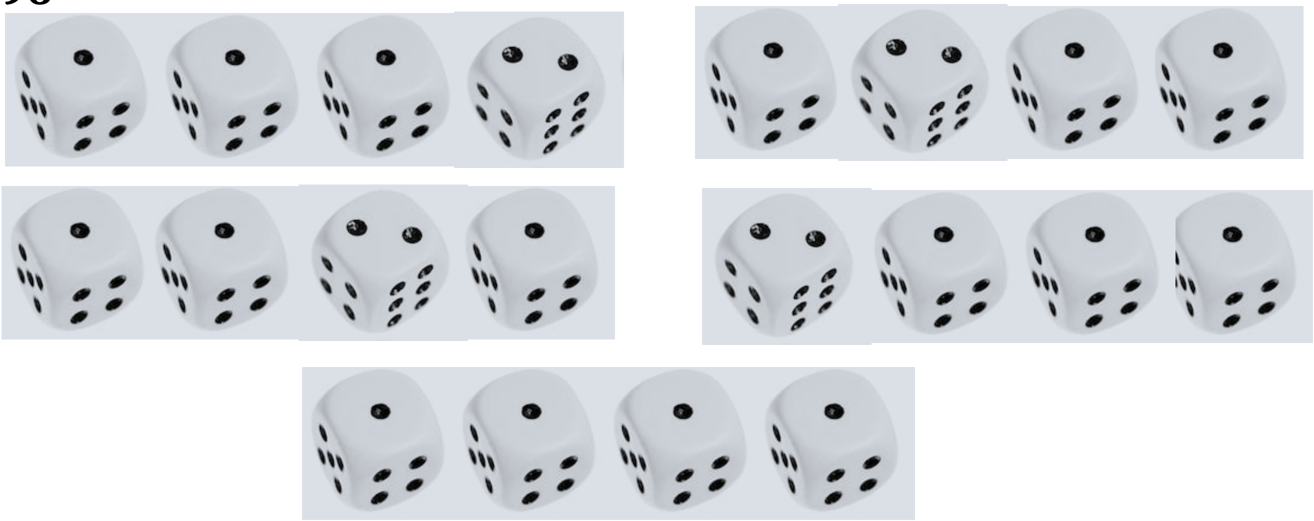
$$1 - \frac{Group\ of\ 3\ female\ students}{Group\ of\ any\ 3\ students} = 1 - \frac{C(11,3)}{C(18,3)} = 1 - \frac{165}{816}$$

The probability of selecting NOT all three females is $\frac{\mathbf{651}}{\mathbf{816}} = \mathbf{0.7978}$

DICE EXAMPLE

- If you roll 4 six sided dice, find the probability that the sum of the dice is greater than 5.
  - Note: Use the compliment:
  - For the sum to be 5 or less, we need to roll all four 1s or roll three 1s and one 2. We can do this 5 ways. The total number of outcomes would be $6 \cdot 6 \cdot 6 \cdot 6 = 6^4 = 1296$

So $1 - \frac{5}{1296} = \frac{\mathbf{1291}}{\mathbf{1396}} = \mathbf{0.9961}$



DICE EXAMPLE

- If you roll 4 six sided dice, find the probability that at least one of the dice has a two showing.
  - Note: Use the compliment
  - For no twos to be showing, each die only has 5 possible outcomes.

$$\mathbf{1} - \frac{\mathbf{5}}{\mathbf{6}} \times \frac{\mathbf{5}}{\mathbf{6}} \times \frac{\mathbf{5}}{\mathbf{6}} \times \frac{\mathbf{5}}{\mathbf{6}}$$

So $1 - \frac{625}{1296} = \frac{\mathbf{671}}{\mathbf{1296}} = \mathbf{0.5177}$

PROBABILITY DEFINITIONS

The **union** of two events E and F, denoted $E \cup F$, is the set of all outcomes in E *or* in F.

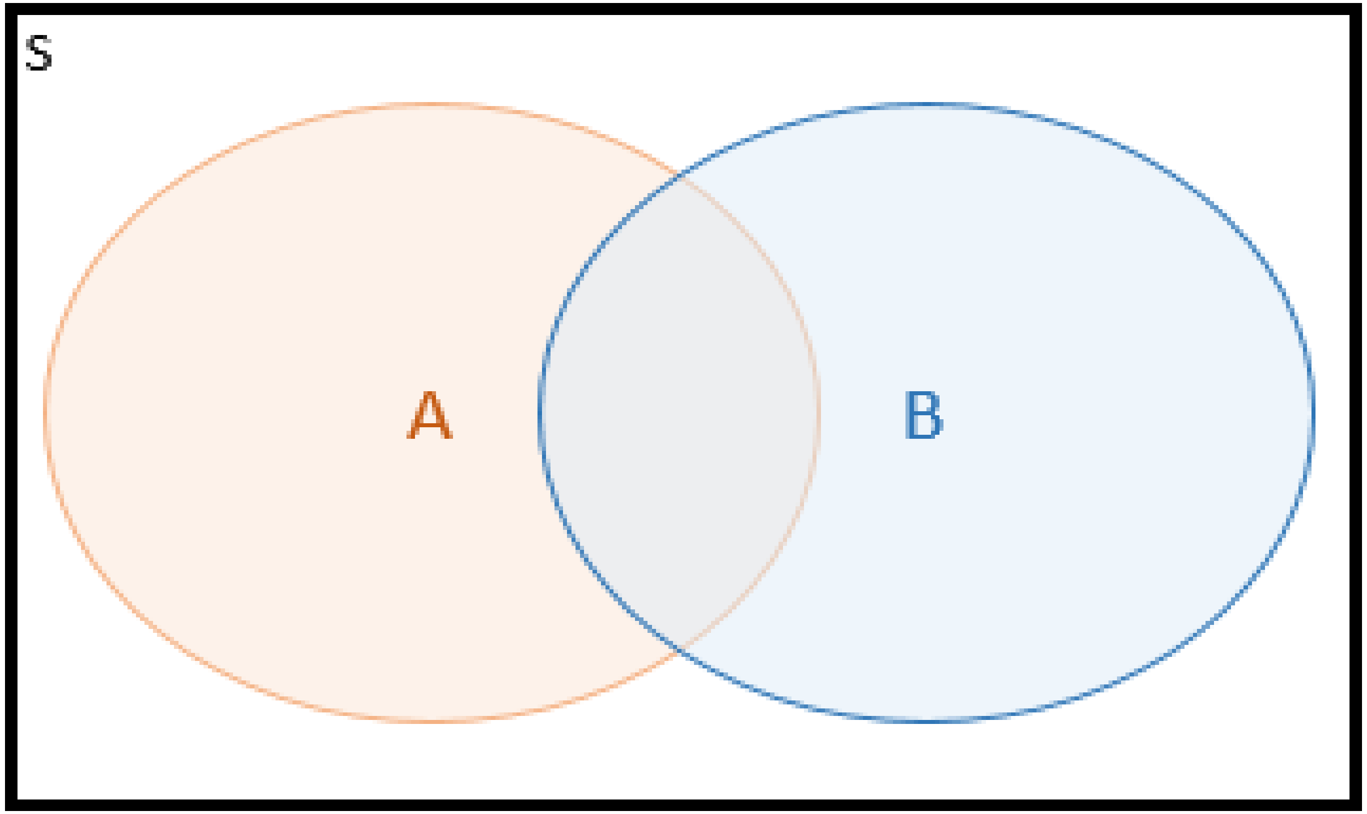
The **intersection** of two events E and F, denoted $E \cap F$, is the set of all outcomes in E *and* in F.

For events E and F, the **probability of E given F** is

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

VENN DIAGRAM

- If $P(A) = 0.42, P(B) = 0.51$, and $P(A \cap \bar{B}) = 0.18$ find the following:
  - $P(A)$
  - $P(B)$
  - $P(A \cap B)$
  - $P(A \cup B)$
  - $P(\bar{A})$
  - $P(\bar{A} \cap B)$
  - $P(\bar{A} \cup B)$
  - $P(\overline{A \cap B})$
  - $P(A | B)$



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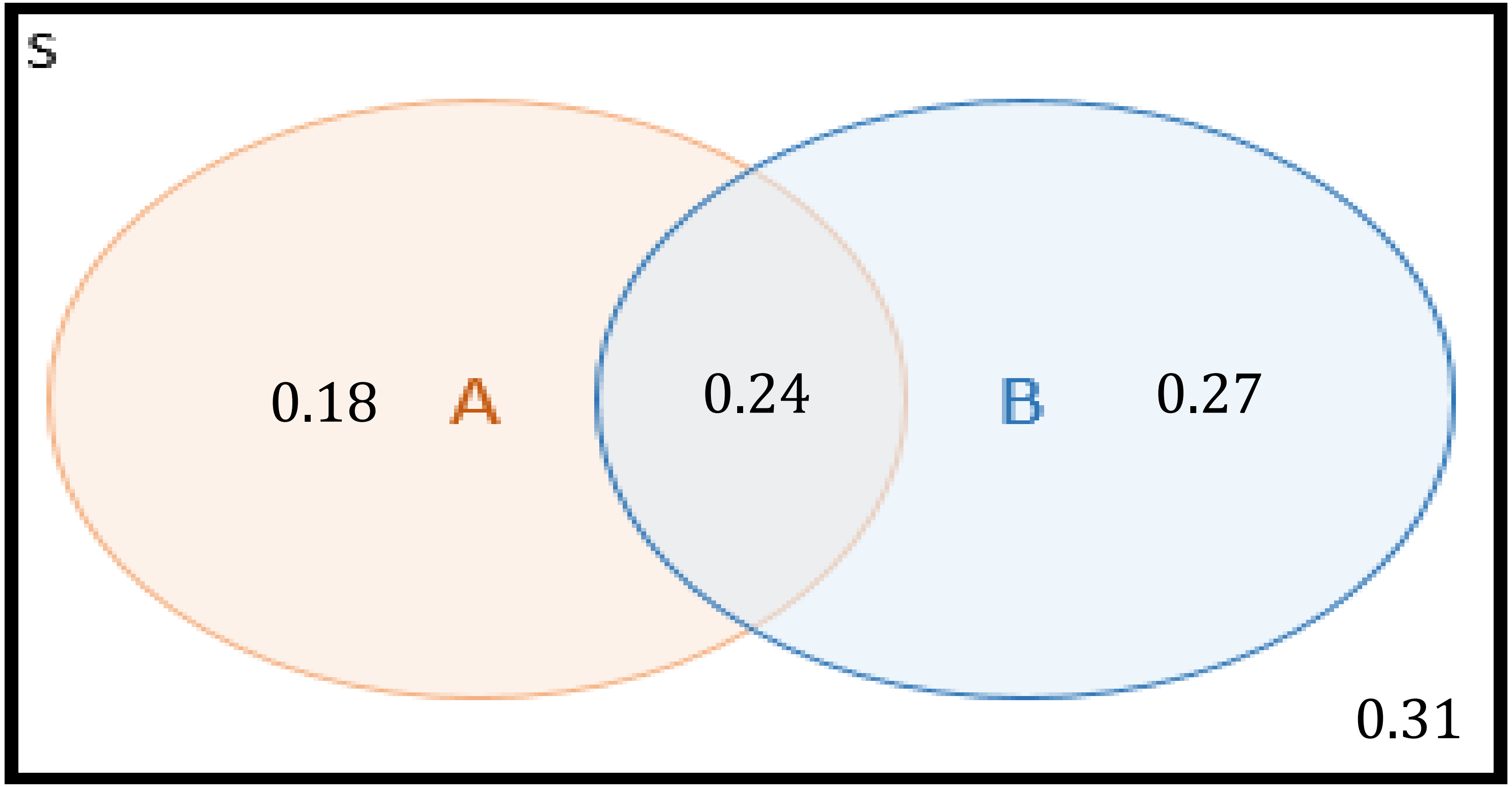
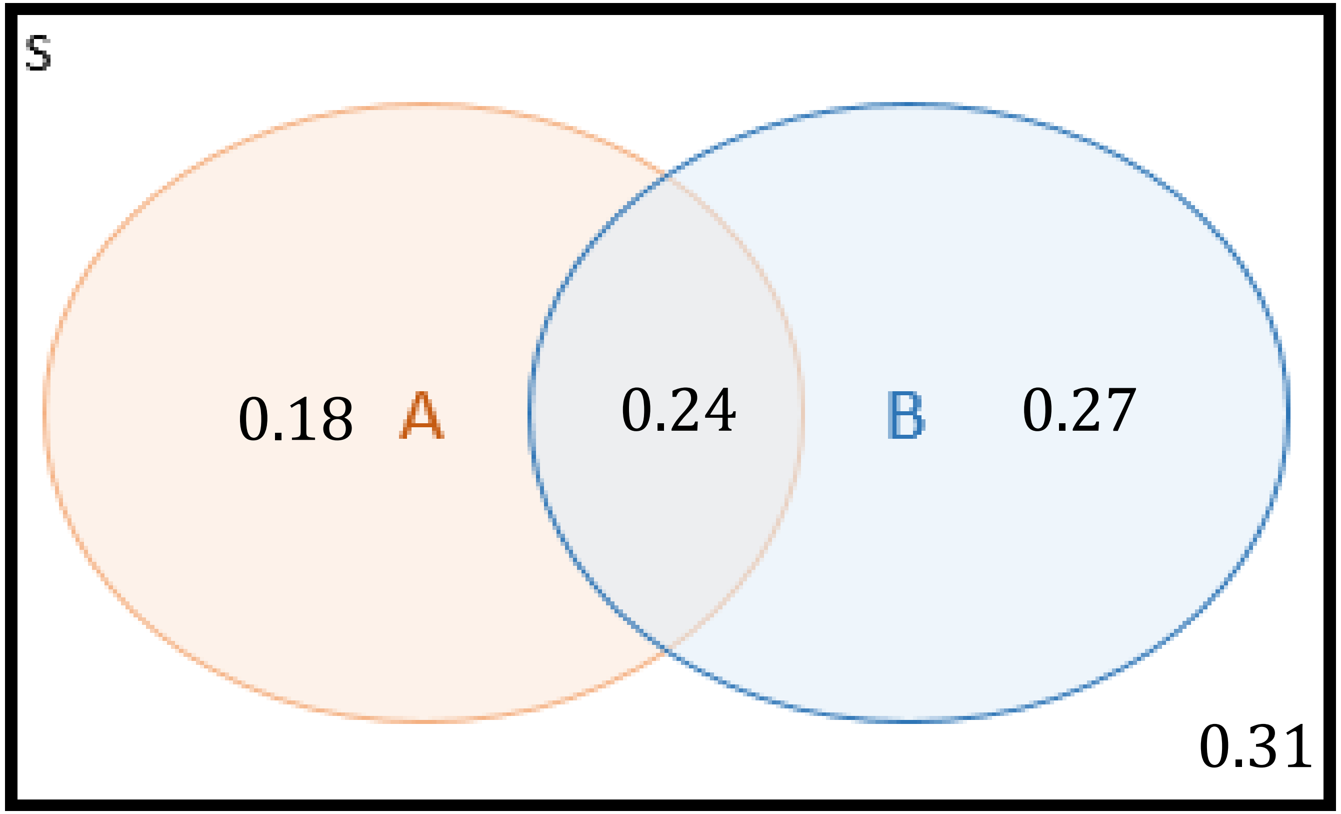


VENN DIAGRAM (FROM PICTURE)

- If $P(A) = 0.42, P(B) = 0.51$, and $P(A \cap \bar{B}) = 0.18$ find the following:
  - $P(A) = \mathbf{0.42}$
  - $P(B) = \mathbf{0.51}$
  - $P(A \cap B) = 0.42 - 0.18 = \mathbf{0.24}$
  - $P(A \cup B) = 0.18 + 0.24 + 0.27 = \mathbf{0.69}$
  - $P(\bar{A}) = 0.27 + 0.31 = \mathbf{0.58}$
  - $P(\bar{A} \cap B) = 0.51 - 0.24 = \mathbf{0.27}$
  - $P(\bar{A} \cup B) = 0.27 + 0.31 + 0.24 = \mathbf{0.82}$
  - $P(\overline{A \cap B}) = 0.18 + 0.27 + 0.31 = \mathbf{0.76}$
  - $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\mathbf{0.24}}{\mathbf{0.51}}$



VENN DIAGRAM (FROM FORMULAS)

- If $P(A) = 0.42, P(B) = 0.51$, and $P(A \cap \bar{B}) = 0.18$ find the following:
  - $P(A) = \mathbf{0.42}$
  - $P(B) = \mathbf{0.51}$
  - $P(A \cap B) = \mathrm{P(A)} - P(A \cap \bar{B}) = .42 - .18 = \mathbf{0.24}$
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .42 + .51 - .24 = \mathbf{0.69}$
  - $P(\bar{A}) = 1 - P(A) = 1 - .42 = \mathbf{0.58}$
  - $P(\bar{A} \cap B) = P(B) - P(A \cap B) = .51 - .24 = \mathbf{0.27}$
  - $P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B) = .58 + .51 - .27 = \mathbf{0.82}$
  - $P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - .24 = \mathbf{0.76}$
  - $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\mathbf{0.24}}{\mathbf{0.51}}$

TREE EXAMPLE

- A diagnostic test for disease X correctly identifies the disease 89% of the time. False positives occur 12%. It is estimated that 2% of the population suffers from disease X. Suppose the test is applied to a random individual from the population.
  - Fill in a probability tree to describe this situation.

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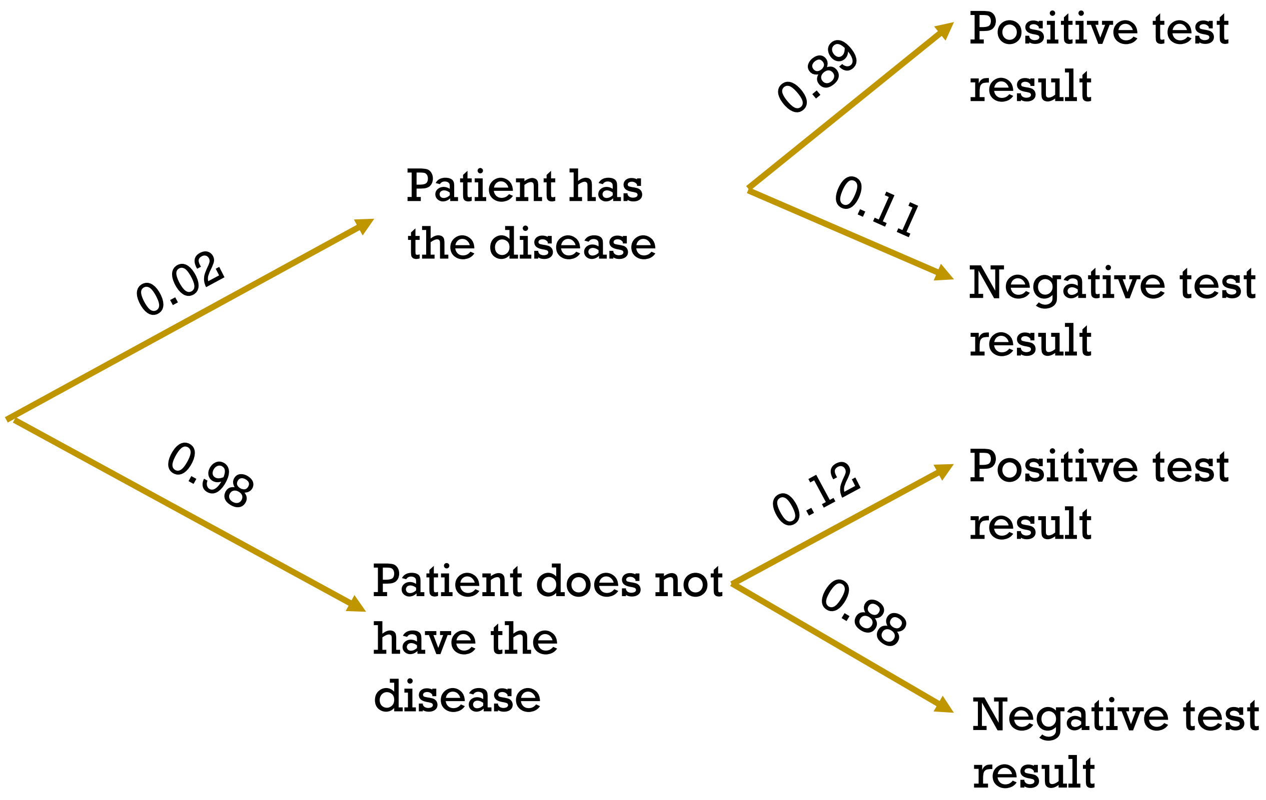

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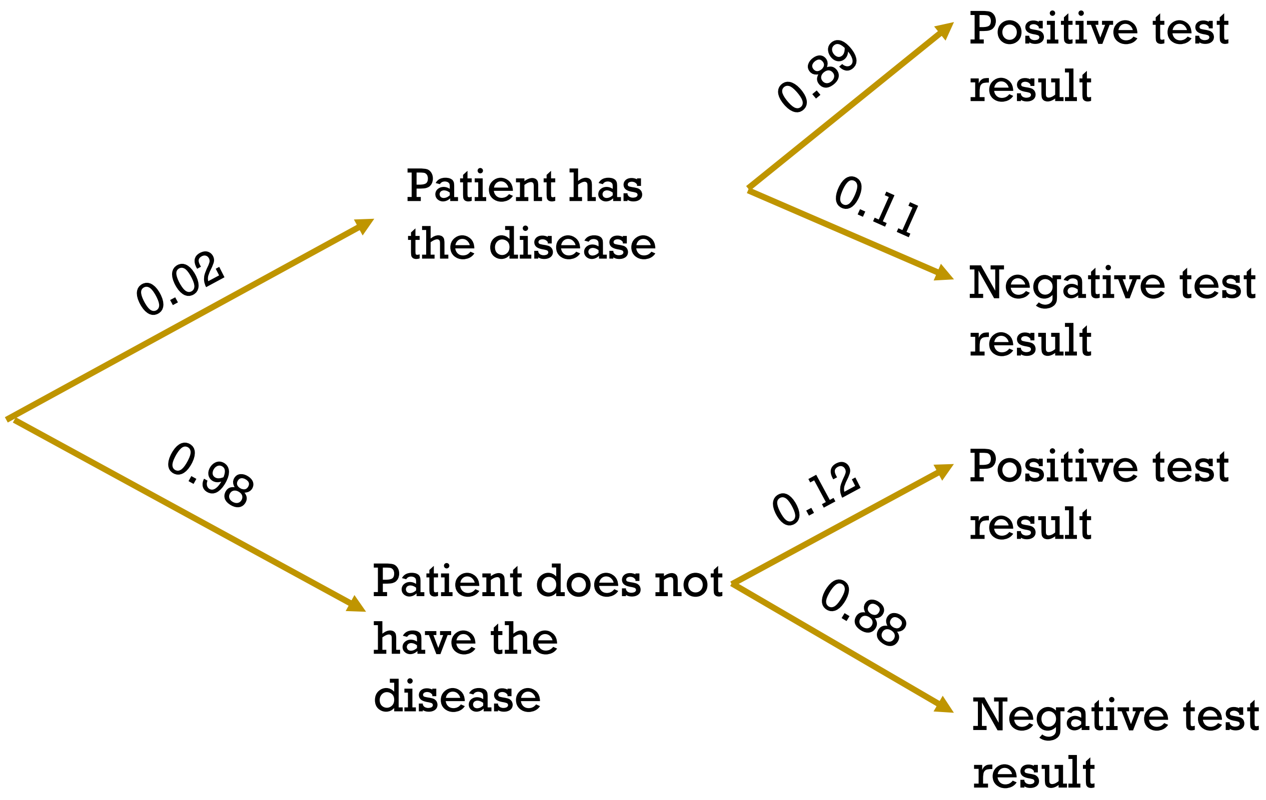

**Note: This includes ALL positive test results, whether they are correct r not*.

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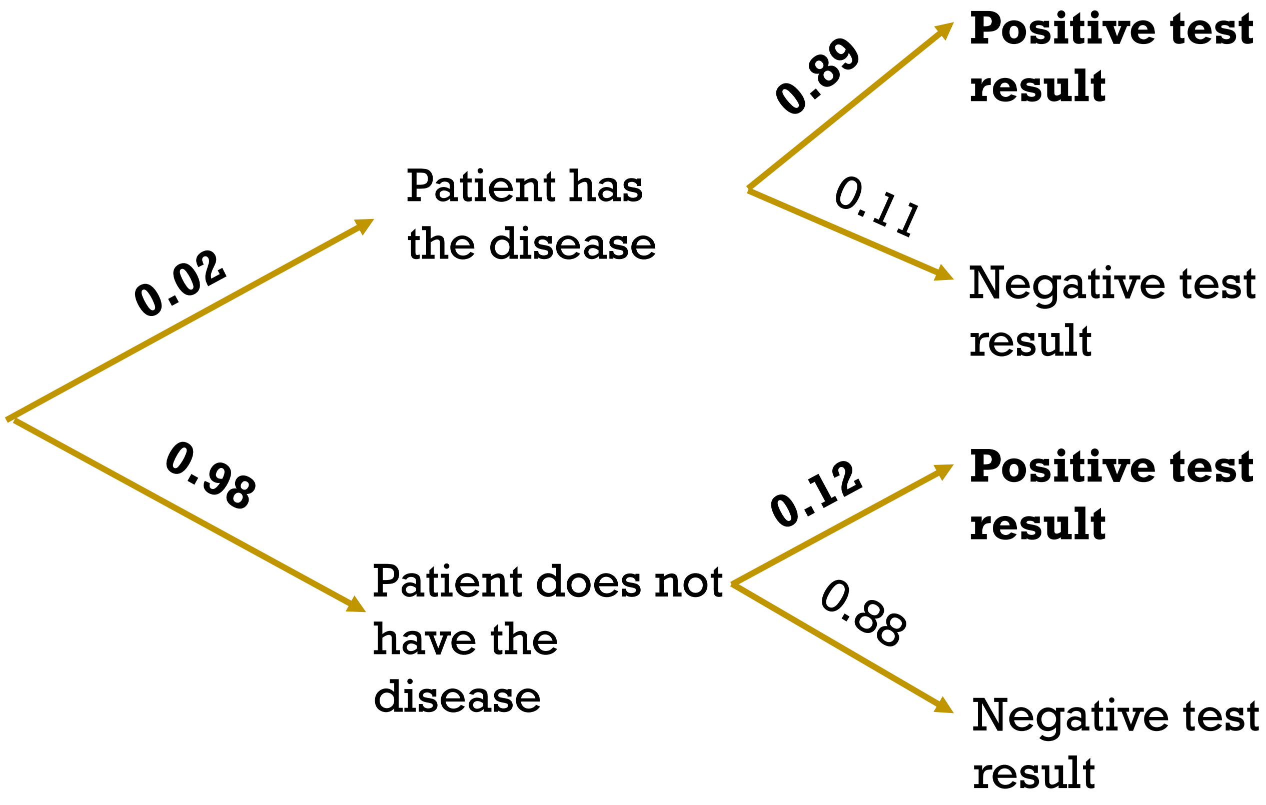
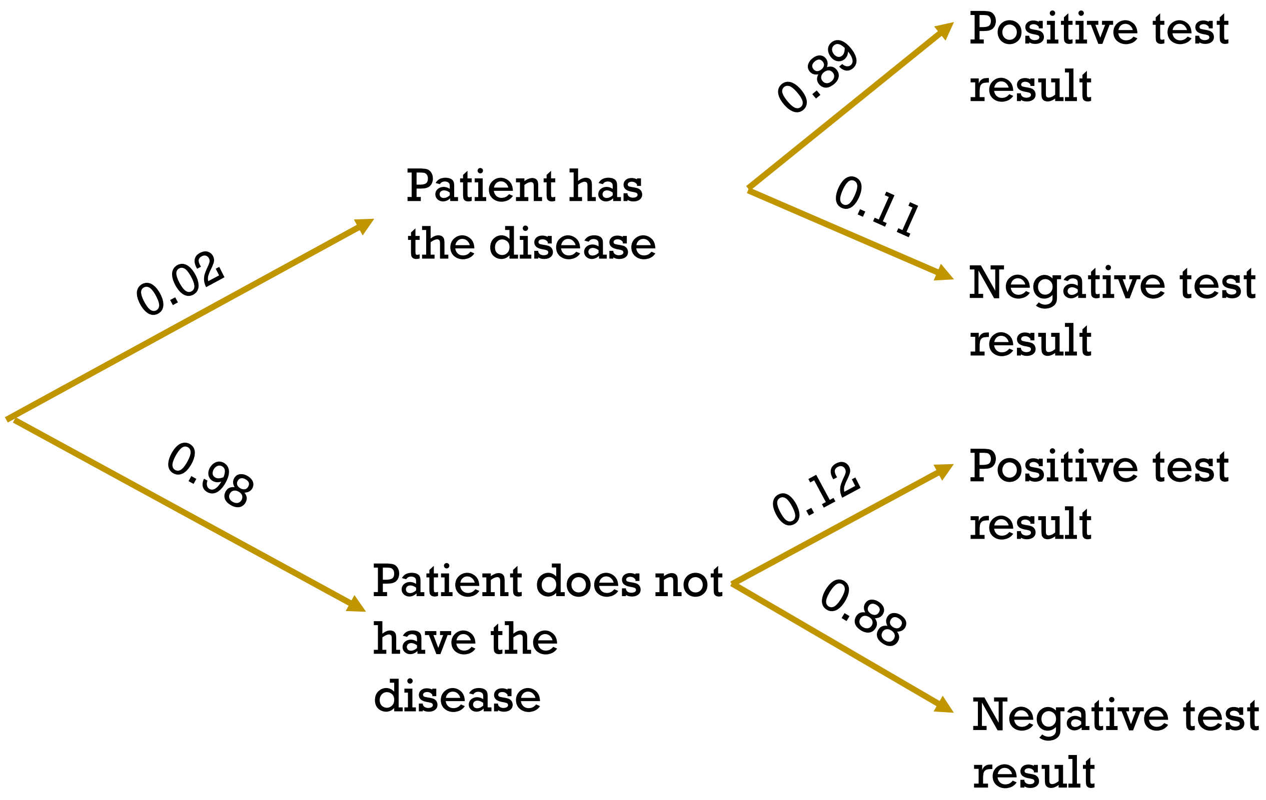


$$(0.02 \cdot 0.89) + (0.98 \cdot 0.12)$$
$$= \mathbf{0.1354\ or\ 13.54\%}$$

**Note: This includes ALL positive test results, whether they are correct or not.*

TREE EXAMPLE

- A diagnostic test for disease X correctly identifies the disease 89% of the time. False positives occur 12%. It is estimated that 2% of the population suffers from disease X. Suppose the test is applied to a random individual from the population.
  - What is the probability that given a positive result, the person has disease X?



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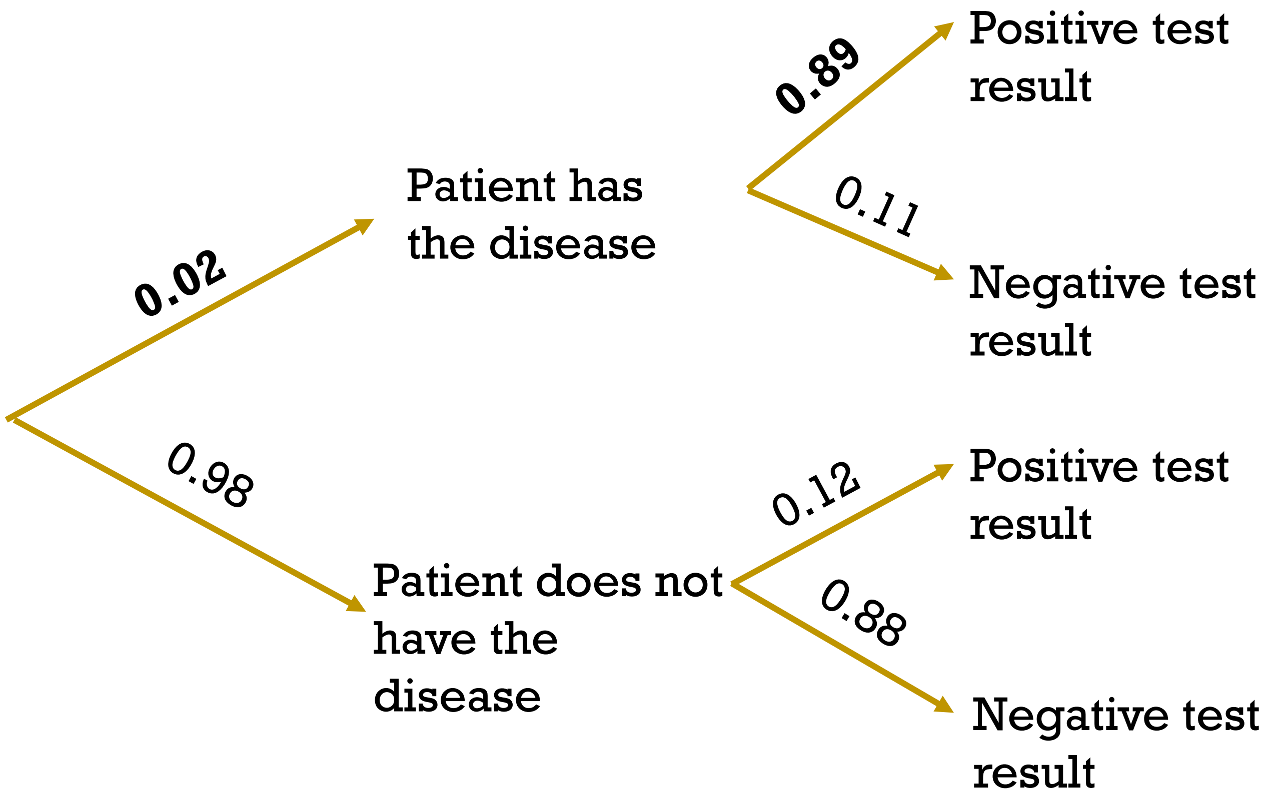

$$\frac{Disease\ and\ Positive\ result}{Positive\ result}$$

$$\frac{(0.02 \cdot 0.89)}{0.1354}$$

$$= \mathbf{0.1315\ or\ 13.15\%}$$

TREE EXAMPLE

- Box A contains 8 red and 10 white marbles. Box B contains 9 red and 5 white marbles. A marble is chosen at random from Box A and its color is recorded. That marble is then placed in Box B and a marble is chosen at random from Box B and its color is recorded.
  - Fill in the probability tree to describe this situation.

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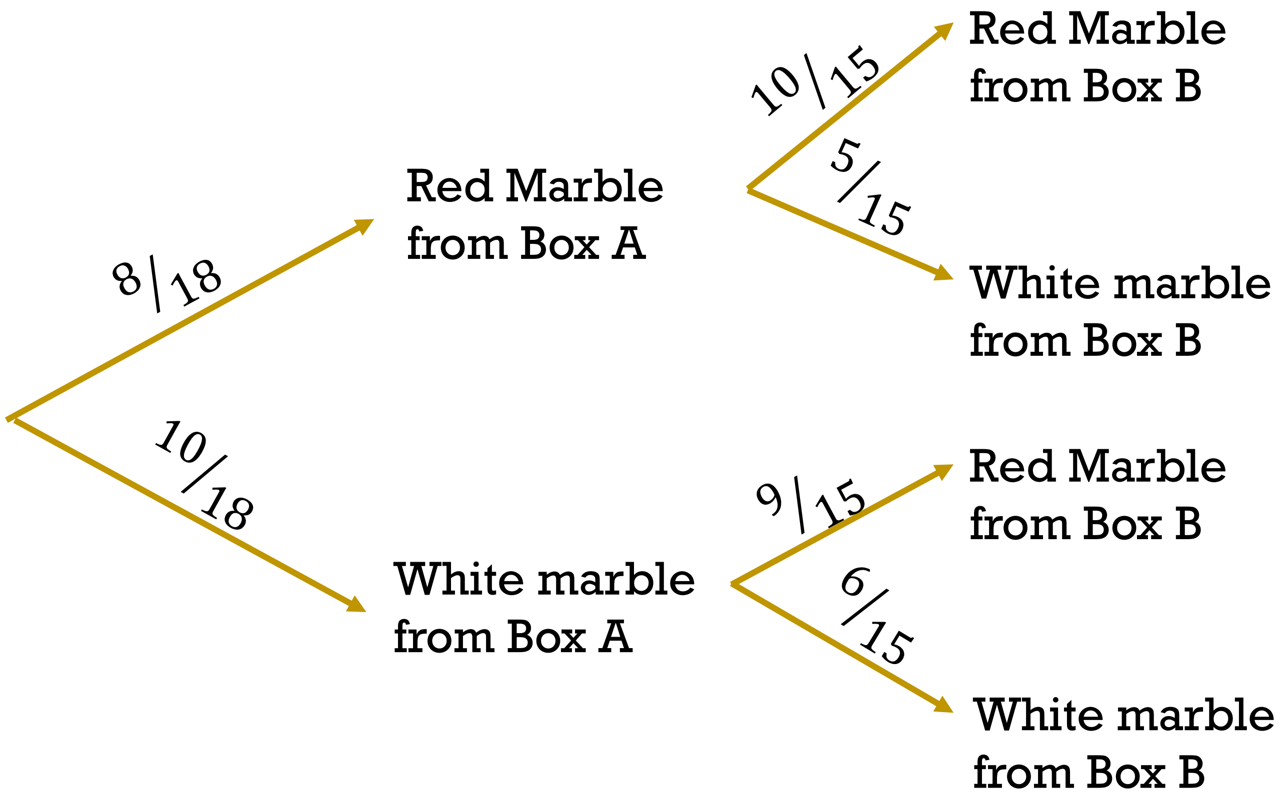

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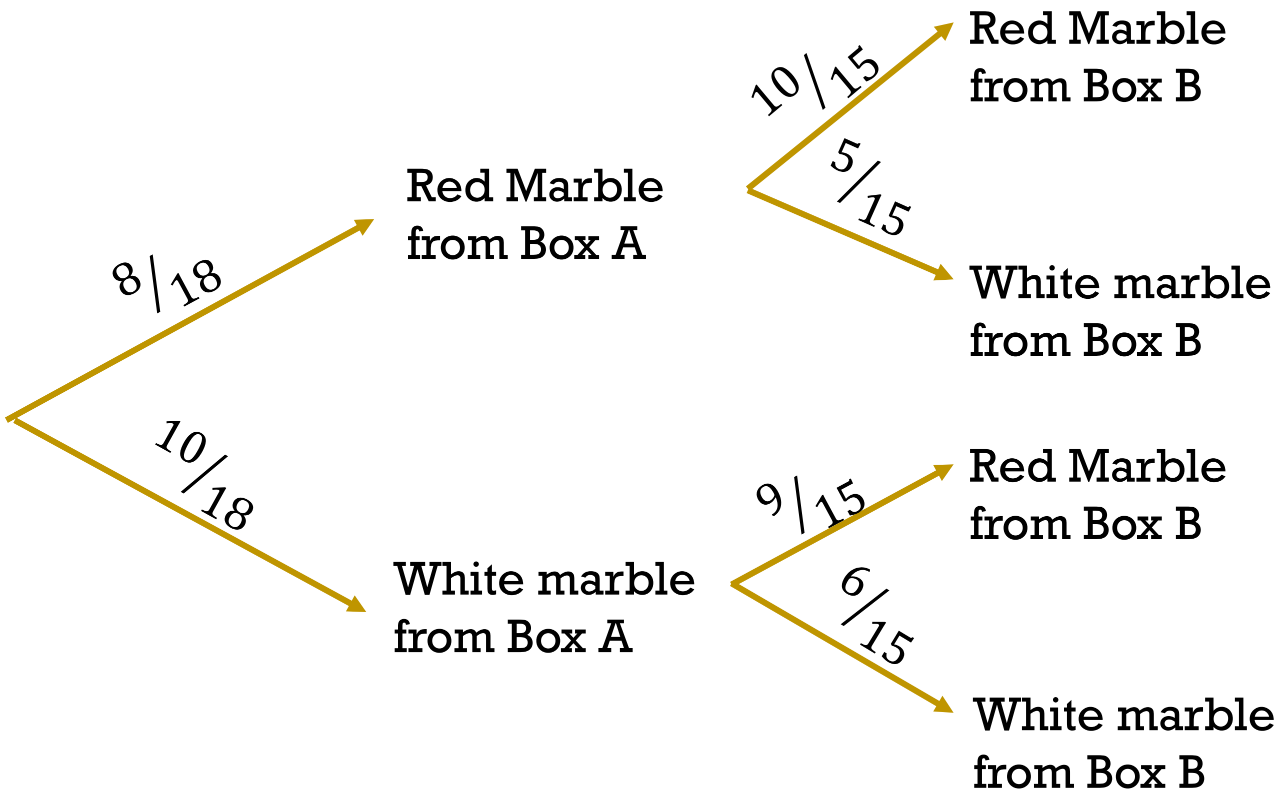
  - Find the probability that a red marble is drawn second.



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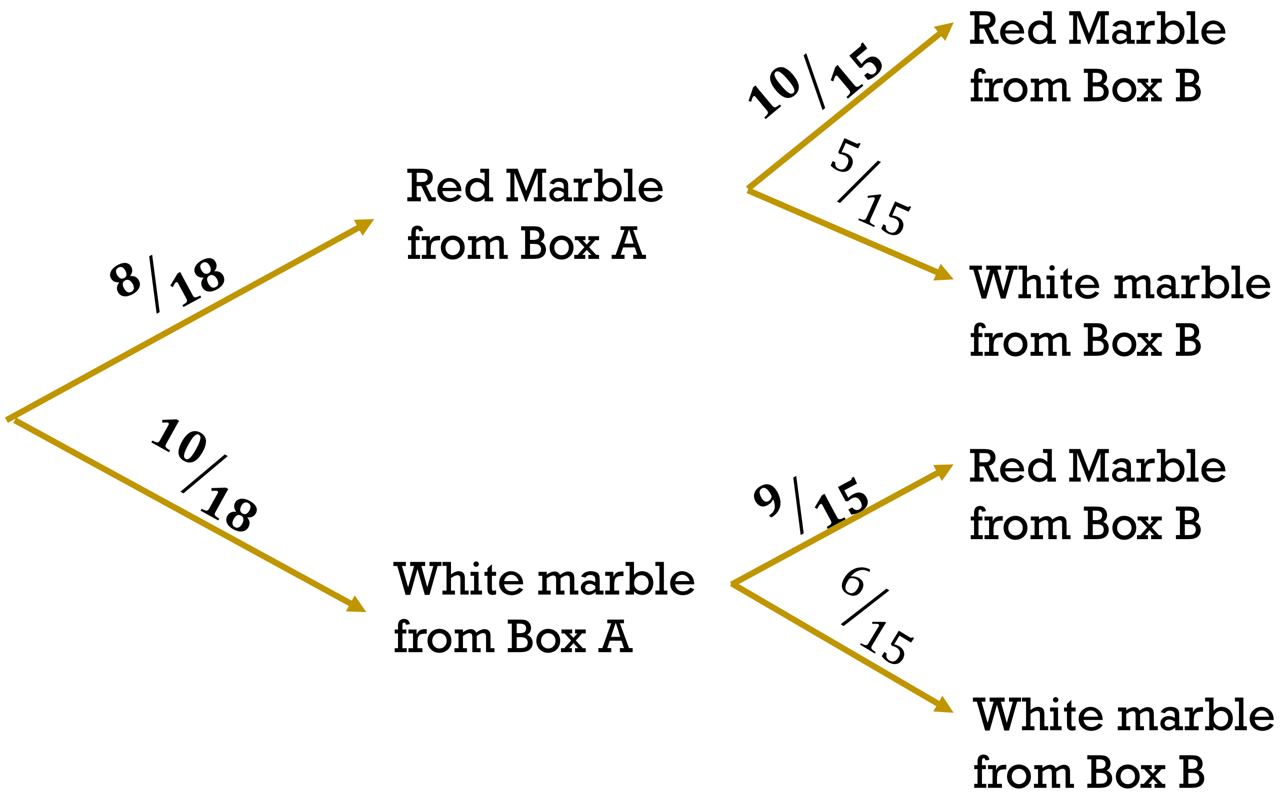

$$P(R_2) = P(RR) + P(WR)$$

$$\frac{8}{18} \cdot \frac{10}{15} + \frac{10}{18} \cdot \frac{9}{15}$$

$$= \mathbf{\frac{17}{27}}$$

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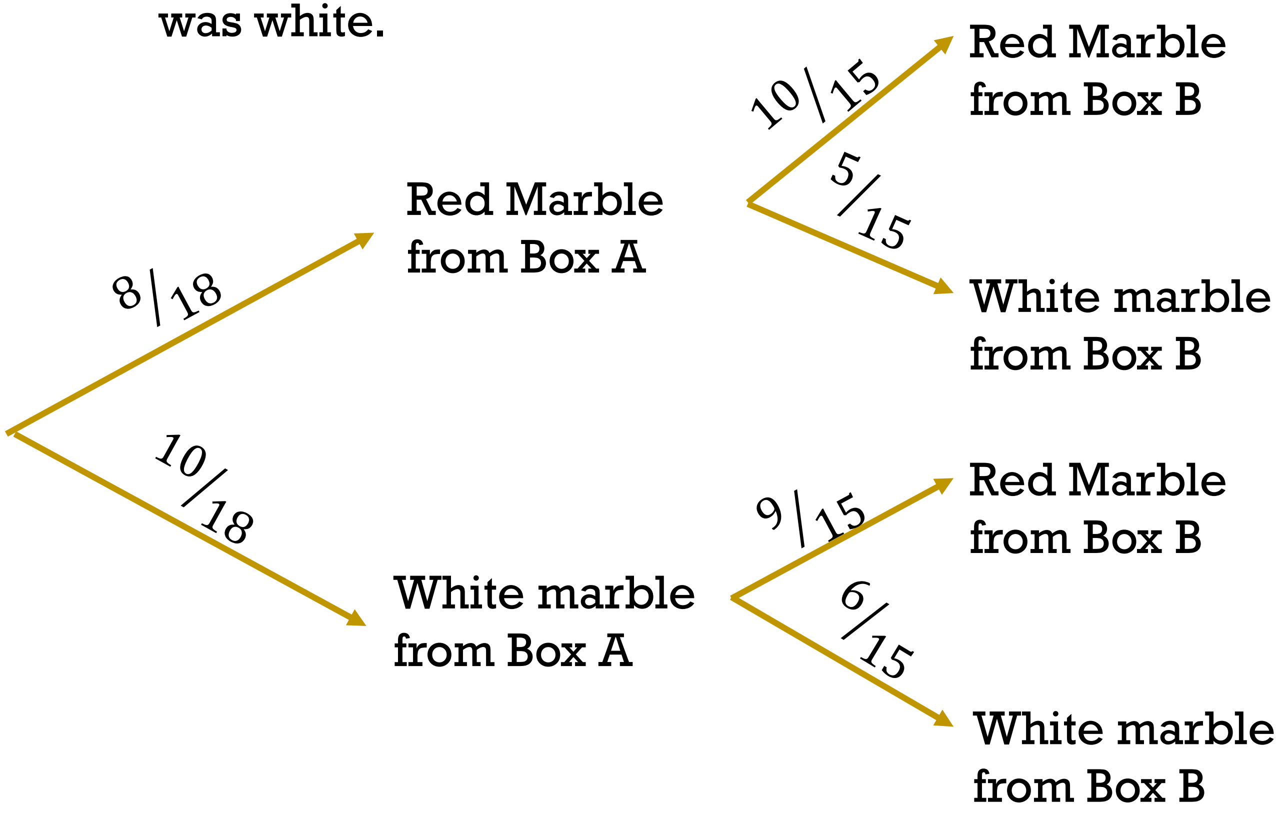
  - Find the probability that you draw a white marble second, given the first marble drawn was white.



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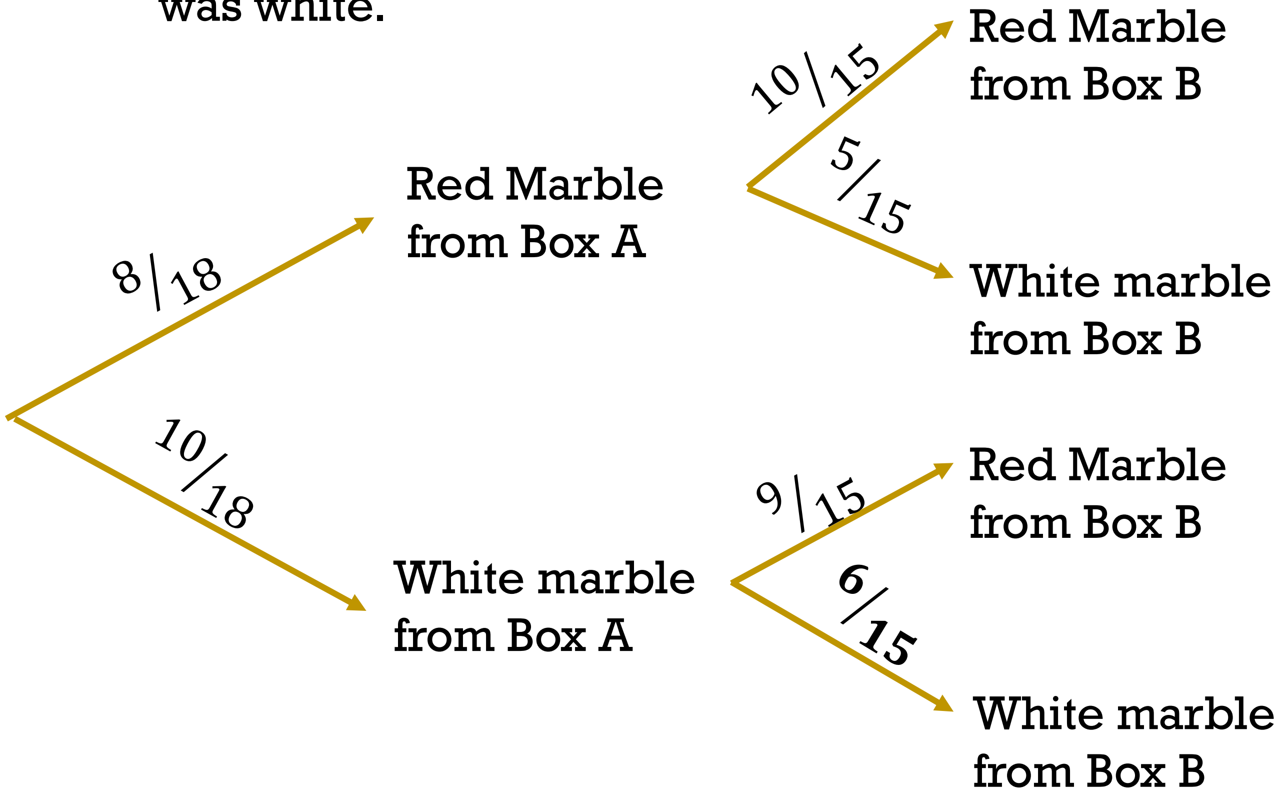

$$\frac{6}{15}$$

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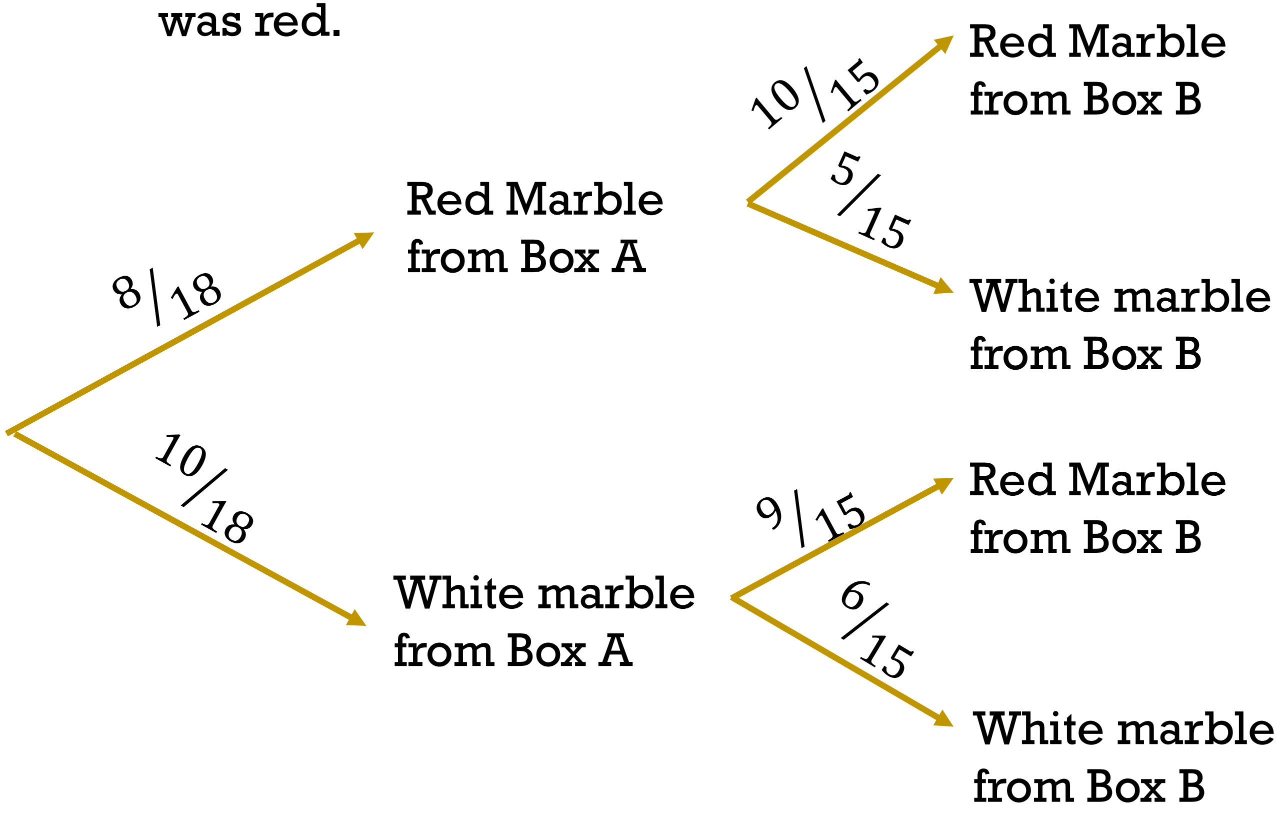

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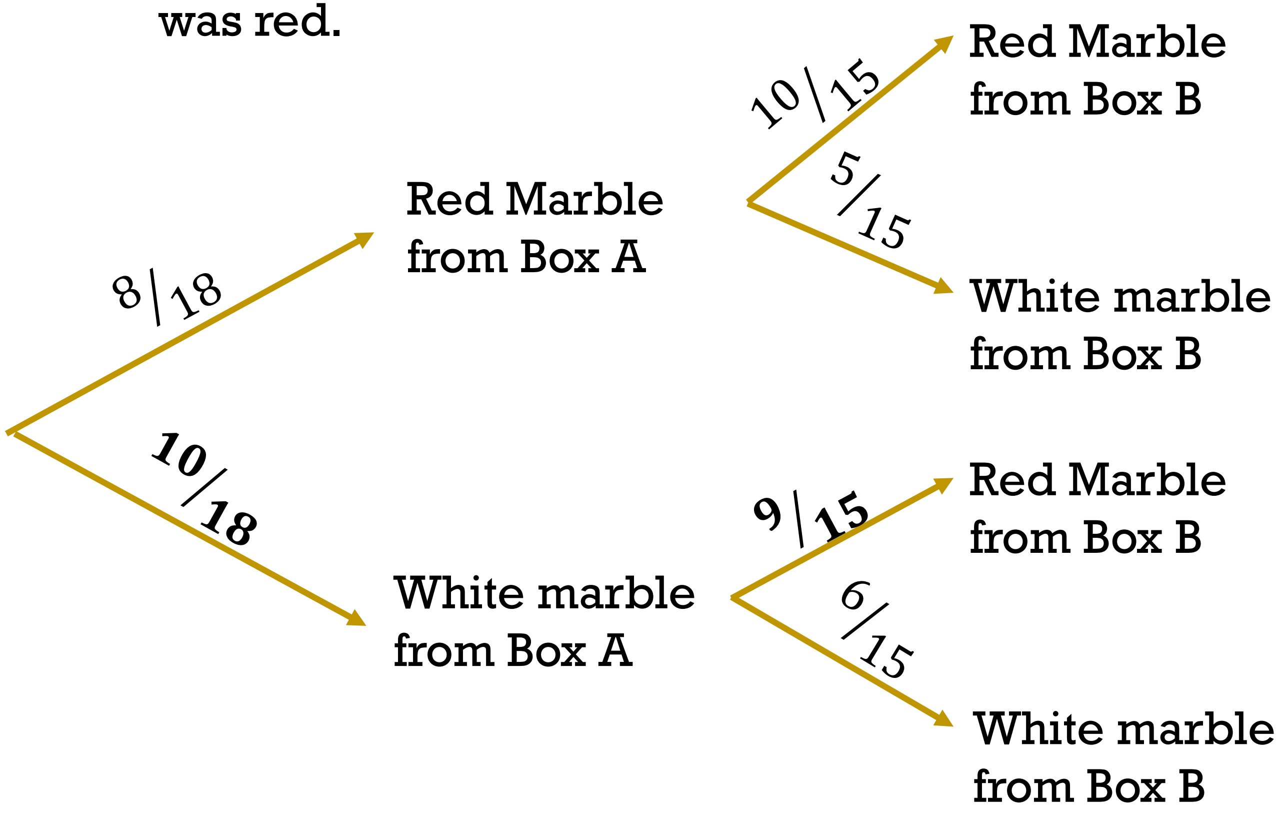

$$P(W_1|R_2) = \frac{P(W_1 \cap R_2)}{P(R_2)}$$

$$\frac{\frac{10}{18} \cdot \frac{9}{15}}{\frac{17}{27}} = \mathbf{\frac{9}{17}}$$

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