MAT110 REVIEW

Finance

PERCENTAGE EXAMPLE

- On Monday gas was $2.39 per gallon. On Wednesday gas prices decreased 6%. On Friday gas prices increased 7.5%. What was the price of a gallon of gas on Friday?

Wednesday: $2.39 - 0.06(2.39) = 2.25$

Friday: $2.25 + 0.075(2.25) = 2.42$

Gas was $2.42 a gallon on Friday.

SIMPLE INTEREST

- **Interest** is the money that one person (a borrower) pays to another (a lender) to use the lender's money.
- The amount you deposit in a bank account is called the **principal**.
- The bank specifies an **interest rate** for that account as a percentage of your deposit.

$$\boldsymbol{I = Prt}$$

Or

$$\boldsymbol{A = P(1 + rt)}$$

EXAMPLE

- If $500 is deposited in a bank account paying 6% simple interest, how much interest will the deposit earn in 4 years?

P = 500 (principal)

r = 0.06 (interest rate)

t = 4 (time in years)

$$I = Prt$$

$$\mathrm{I} = 500 \times 0.06 \times 4$$

$$\mathrm{I} = 120$$

The deposit will earn $120 in interest

EXAMPLE

- If \$700 is invested in an account earning simple interest and the amount in the account after 18 months is \$836.50, what is the annual interest rate?

$$A = P(1 + rt)$$

$P = 700$ (principal)

$A = 836.50$ (final amount in the account)

$t = \frac{18}{12}$ (time in years)

$$836.5 = 700\left(1 + r\frac{18}{12}\right)$$

$$\frac{836.5}{700} = 1 + r\frac{18}{12}$$

$$\frac{836.5}{700} - 1 = r\frac{18}{12}$$

$$\frac{\frac{836.5}{700} - 1}{\frac{18}{12}} = r$$

The interest rate is 0.13 or 13%

COMPOUND INTEREST

- Interest that is paid on principal plus previously earned interest is called **compound interest.**
- If the interest is added each month(12 times per year), we say that the interest is **compounded monthly**.
  - We can also have interest compounded annually(once per year), quarterly(4 times per year), daily(365 times per year), weekly(52 times per year), or continuously.

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$A = Pe^{rt}$$

EXAMPLE

- How much money should be invested in an account earning 4.5% interest compounded continuously if Griff wants to have $8500 in the account after 5 years?
- Use the formula: $A = Pe^{rt}$

$$8500 = Pe^{0.045(5)}$$

$$\frac{8500}{e^{0.045(5)}} = P$$

$$6787.387859 = P$$

Griff should invest $6787.39.

EXAMPLE

- Max invests \$7500 into an account earning 9% interest compounded quarterly. How long will it take to double his money?
- Use the formula: $A = P\left(1+\frac{r}{m}\right)^{mt}$

$$15000 = 7500\left(1+\frac{0.09}{4}\right)^{4t}$$

$$2 = \left(1+\frac{0.09}{4}\right)^{4t}$$

$$\ln(2) = \ln\left(1+\frac{0.09}{4}\right)^{4t}$$

$$\ln(2) = 4t\ln\left(1+\frac{0.09}{4}\right)$$

$$\frac{\ln(2)}{4\ln\left(1+\frac{0.09}{4}\right)} = t$$

$$\boldsymbol{t = 7.78796}\ \textbf{years}$$

EXAMPLE

- Griff invests $1600 into an account earning 7.25% interest compounded continuously. How long will it take for the amount in the account to reach $2400?
- Use the formula: $A = Pe^{rt}$

$$2400 = 1600e^{0.0725t}$$

$$\frac{2400}{1600} = e^{0.0725t}$$

$$\ln\left(\frac{2400}{1600}\right) = \ln(e^{0.0725t})$$

Note: $ln(e) = 1$

$$\ln\left(\frac{2400}{1600}\right) = 0.0725\text{t} \cdot \ln(e)$$

$$\frac{\ln\left(\frac{2400}{1600}\right)}{0.0725} = t$$

$$\boldsymbol{t = 5.592622 \text{ years}}$$

EXAMPLE

- Griff invests \$500 in an account earning interest compounded monthly. Determine the rate if there was \$840 in the account 6 years later.
- Use the formula: $A = P\left(1 + \frac{r}{m}\right)^{mt}$

$$840 = 500\left(1 + \frac{r}{12}\right)^{12(6)}$$

$$\frac{840}{500} = \left(1 + \frac{r}{12}\right)^{72}$$

Note: $(x^3)^{\frac{1}{3}} = x$

$$\left(\frac{840}{500}\right)^{\frac{1}{72}} = 1 + \frac{r}{12}$$

$$\left(\frac{840}{500}\right)^{\frac{1}{72}} - 1 = \frac{r}{12}$$

$$12\left(\left(\frac{840}{500}\right)^{\frac{1}{72}} - 1\right) = r$$

$$\boldsymbol{r = 0.086777895}$$

The interest rate is 8.678%

EXAMPLE

- Determine the interest earned after 15 years if $5400 is invested in each of the following accounts. Which account is a better investment?

Account earning 5.4% compounded monthly

$$A = 5400\left(1 + \frac{0.054}{12}\right)^{12(15)}$$

$$A = 12116.67$$

$$I = 12116.67 - 5400$$

$$\boldsymbol{I = 6716.67}$$

Account earning 5.9% simple interest

$$I = 5400(0.059)(15)$$

$$\boldsymbol{I = 4779}$$

ANNUITIES

- An **annuity** is an interest-bearing account into which we make a series of payments of the same size.
- If one payment is made at the *end* of every compounding period, the annuity is called an **ordinary annuity**.
- The **future value of an annuity** is the amount in the account, including interest, after making all payments.

$$A = R\left[\frac{\left(1+\frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}}\right]$$

Note: A sinking fun is a special kind of annuity.

EXAMPLE

- A payment of $50 is made at the end of each month into an account paying a 6% annual interest rate, compounded monthly. How much will be in that account after 3 years?
  - Use the formula: $A = R\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right]$

$$A = 50\left[\frac{\left(1+\frac{0.06}{12}\right)^{12(3)}-1}{\frac{0.06}{12}}\right]$$

$$A = 1966.805248$$

There will be $1966.81 in the account.

EXAMPLE

- A payment of $50 is made at the end of each month into an account paying a 6% annual interest rate, compounded monthly. How much interest is earned at the end of 3 years?

  - Interest in an Annuity = Final Amount in the Account – Total Paid into the Account

Final Amount in the Account: 1966.81

Total Paid in the Account: $50 \cdot 12 \cdot 3 = 1800$

Interest Earned: $1966.81 - 1800 = 166.81$

You will earn $166.81 in interest.

EXAMPLE

- Assume that you wish to save $2,800 in a sinking fund in 2 years. If you invest your money in an ordinary annuity earning an annual interest rate of 4.5%, compounded monthly, what should your monthly payment be?
  - Use the formula: $A = R\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right]$

$$2800 = R\left[\frac{\left(1+\frac{0.045}{12}\right)^{12(2)}-1}{\frac{0.045}{12}}\right]$$

$$2800 \div \left[\frac{\left(1+\frac{0.045}{12}\right)^{12(2)}-1}{\frac{0.045}{12}}\right] = R$$

$$111.7138724 = R$$

The monthly payment should be $111.71

EXAMPLE

- Suppose you have decided to retire as soon as you have saved $1,000,000. Your plan is to put $200 each month into an ordinary annuity that pays an annual interest rate of 8%. In how many years will you be able to retire?
  - Use the formula: $A = R\left[\frac{\left(1+\frac{r}{m}\right)^{m\cdot t}-1}{\frac{r}{m}}\right]$

$$1000000 = 200\left[\frac{\left(1+\frac{.08}{12}\right)^{12\cdot t}-1}{\frac{.08}{12}}\right]$$

$$\frac{1000000}{200} = \left[\frac{\left(1+\frac{.08}{12}\right)^{12\cdot t}-1}{\frac{.08}{12}}\right]$$

$$\frac{1000000}{200}\cdot\frac{.08}{12} = \left(1+\frac{.08}{12}\right)^{12\cdot t}-1$$

$$\frac{1000000}{200}\cdot\frac{.08}{12}+1 = \left(1+\frac{.08}{12}\right)^{12\cdot t}$$

$$\ln\left(\frac{1000000}{200}\cdot\frac{.08}{12}+1\right) = 12\mathrm{t}\ln\left(1+\frac{.08}{12}\right)$$

$$\frac{\ln\left(\frac{1000000}{200}\cdot\frac{.08}{12}+1\right)}{12\ln\left(1+\frac{.08}{12}\right)} = t$$

$$t = 44.34863377$$

So you will be able to retire in 44.3486 years

AMORTIZATION

- The process of paying off a loan (plus interest) by making a series of regular, equal payments is called **amortization**, and such a loan is called an **amortized loan**.
  - *Amortized loans include mortgages, car loans, students payments, etc.*

$$\boldsymbol{P = R\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right]}$$

EXAMPLE

- An amortized loan of $10,000 is made to pay off a car in 4 years. If the yearly interest rate is 18%, what is your monthly payment?
  - Use the formula: $P = R\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right]$

$$10000 = R\left[\frac{1-\left(1+\frac{0.18}{12}\right)^{-12\cdot 4}}{\frac{0.18}{12}}\right]$$

$$10000 \div \left[\frac{1-\left(1+\frac{0.18}{12}\right)^{-12\cdot 4}}{\frac{0.18}{12}}\right] = R$$

$$R = 293.749996$$

The monthly payment should be $293.75

EXAMPLE

- What is the maximum amount you can borrow with a 15-year mortgage if you can pay $550 a month and have been offered a 4.68% interest rate, compounded monthly?
  - Use the formula: $P = R\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right]$

$$\mathrm{P} = 550\left[\frac{1-\left(1+\frac{0.0468}{12}\right)^{-12\cdot 15}}{\frac{0.0468}{12}}\right]$$

$$P = 71038.81139$$

You can borrow $71038.81

EXAMPLE

- How much will you pay in interest over the life of the loan if you make the minimum monthly payment each month?
  - *Interest on a Loan = Total Paid – Amount Borrow*

Total Paid: $550 \cdot 12 \cdot 15 = 99000$

Amount Borrowed: 71038.81

Interest: $99000 - 71038.81 = 27961.19$

You will pay $27961.19 in interest

EXAMPLE

- Suppose you have a 30-year mortgage for \$100,000 at an annual interest rate of 9%. After 10 years, you refinance. How much remains to be paid on your mortgage?
  - Use the formula: $P = R\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right]$ to find the monthly payments first.

$$100000 = R\left[\frac{1-\left(1+\frac{0.09}{12}\right)^{-12(30)}}{\frac{0.09}{12}}\right]$$

$$100000 \div \left[\frac{1-\left(1+\frac{0.09}{12}\right)^{-12(30)}}{\frac{0.09}{12}}\right] = R$$

$$R = 804.6226168$$

The monthly payment is \$804.62

EXAMPLE

- Suppose you have a 30-year mortgage for $100,000 at an annual interest rate of 9%. After 10 years, you refinance. How much remains to be paid on your mortgage?
  - The unpaid balance on the loan is: $U = R\left[\frac{1-\left(1+\frac{r}{m}\right)^{-N}}{\frac{r}{m}}\right]$ where N is the number of payments left on the loan.

$$U = 804.62\left[\frac{1-\left(1+\frac{0.09}{12}\right)^{-12(20)}}{\frac{0.09}{12}}\right]$$

$$U = 89429.45291$$

You still owe $89429.45 on this mortgage.

EXAMPLE

- The remaining 20 years is financed at an annual interest rate of 7.2%. What are the new monthly payments?
  - Use the formula: $P = R\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right]$

$$89429.45 = R\left[\frac{1-\left(1+\frac{0.072}{12}\right)^{-12(20)}}{\frac{0.072}{12}}\right]$$

$$89429.45 \div \left[\frac{1-\left(1+\frac{0.072}{12}\right)^{-12(20)}}{\frac{0.072}{12}}\right] = R$$

$$R = 704.1221478$$

The new monthly payment will be $704.12

EXAMPLE

- How much will you save in interest in 20 years by paying the lower rate?

9% Interest Rate

- Total amount paid over 20 years:
  $804.62 \cdot 12 \cdot 20 = 193108.80$

7.2% Interest Rate

- Total amount paid over 20 years:
  $704.12 \cdot 12 \cdot 20 = 168988.80$

Amount Saved: $193108.80 - 168988.80 = 24120$

You will save $24,120

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